11) Variable acceleration

11.1) Functions of time

Worked example	Your turn
A body moves in a straight line, such that its displacement, s metres, from a point O at time t seconds, is given by $s = 5t^3 - 2t, t > 0$ Find: a) s when $t = 3$ b) The time taken for the particle to return to O	A body moves in a straight line, such that its displacement, s metres, from a point O at time t seconds, is given by $s = 2t^3 - 3t, t > 0$ Find: a) s when $t = 2$ b) The time taken for the particle to return to O a) $10\ m$ b) $\sqrt{\frac{3}{2}}\ s = 1.2\ s$ (2 sf)

Worked example	Your turn
A train travels along a straight track, leaving the start of the track at time $t = 0$. It then returns to the start of the track. The distance, s metres, from the start of the track at time t seconds is modelled by: $$s = 8t^2 - 5t^3, \quad 0 \leq t \leq 1.6$$ Explain the restriction $0 \leq t \leq 1.6$	A train travels along a straight track, leaving the start of the track at time $t = 0$. It then returns to the start of the track. The distance, s metres, from the start of the track at time t seconds is modelled by: $$s = 4t^2 - t^3, 0 \leq t \leq 4$$ Explain the restriction $0 \leq t \leq 4$ s is the distance from the start of the track: $s \geq 0$ $$4t^2 - t^3 \geq 0$$ $$t^2(4 - t) \geq 0$$ $t^2 \geq 0$ for all t and $(4 - t) < 0$ for all $t > 4$. So $t^2(4 - t)$ is only non-negative for $t \leq 4$ Motion begins at $t = 0$, hence $t \geq 0$ Hence $0 \leq t \leq 4$

Worked example	Your turn
A body moves in a straight line such that its velocity, v ms^{-1}, at time t seconds is given by $v = 3t^2 - 24t + 36$. Find (a) The initial velocity (b) The values of t when the body is instantaneously at rest. (c) The value of t when the velocity is 63 ms^{-1}. (d) The greatest speed of the body in the interval $0 \leq t \leq 7$.	A body moves in a straight line such that its velocity, v ms^{-1}, at time t seconds is given by $v = 2t^2 - 16t + 24$. Find (a) The initial velocity (b) The values of t when the body is instantaneously at rest. (c) The value of t when the velocity is 64 ms^{-1}. (d) The greatest speed of the body in the interval $0 \leq t \leq 5$. a) $24\ ms^{-1}$ b) $t = 2, t = 6$ c) $t = 10$ d) $24\ ms^{-1}$

11.2) Using differentiation

Worked example

A particle P is moving on the x-axis.
At time t seconds, the displacement x metres from O is given by

$$x = 3t^4 - 96t + 7$$

Find:

(a) the velocity of P when $t = 5$

(b) The value of t when P is instantaneously at rest

(c) The acceleration of P when $t = 0.5$

Your turn

A particle P is moving on the x-axis.
At time t seconds, the displacement x metres from O is given by

$$x = t^4 - 32t + 14$$

Find:

(a) the velocity of P when $t = 3$

(b) The value of t when P is instantaneously at rest

(c) The acceleration of P when $t = 1.5$

a) $76\ ms^{-1}$

b) $t = 2$

c) $27\ ms^{-2}$

Worked example

A particle P is moving on the x-axis.
At time t seconds, the displacement x metres from O is given by

$$x = \frac{1}{3}t^3 - \frac{7}{2}t^2 + 12t + 15$$

Find the distance between the two points at which the particle is at rest.

Your turn

A particle P is moving on the x-axis.
At time t seconds, the displacement x metres from O is given by

$$x = \frac{1}{3}t^3 - \frac{11}{2}t^2 + 30t + 5$$

Find the distance between the two points at which the particle is at rest.

$0.17\ m$ (2s f)

11.3) Maxima and minima problems

Chapter CONTENTS

Worked example

A child is playing with a yo-yo. The yo-yo leaves the child's hand at time $t = 0$ and travels vertically in a straight line before returning to the child's hand. The distance, s m, of the yo-yo from the child's hand after time t seconds is given by:

$$s = 2.4t - 0.4t^2 - 0.4t^3, \qquad 0 \leq t \leq 2$$

(a) Justify the restriction $0 \leq t \leq 2$

(b) Find the maximum distance of the yo-yo from the child's hand, correct to 3sf.

Your turn

A child is playing with a yo-yo. The yo-yo leaves the child's hand at time $t = 0$ and travels vertically in a straight line before returning to the child's hand. The distance, s m, of the yo-yo from the child's hand after time t seconds is given by:

$$s = 0.6t + 0.4t^2 - 0.2t^3, \qquad 0 \leq t \leq 3$$

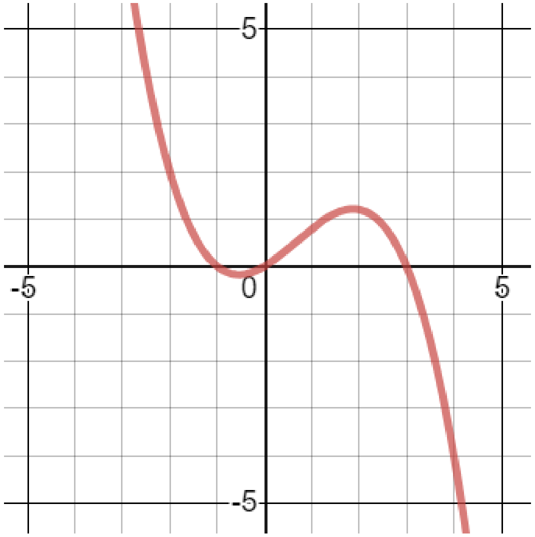
(a) Justify the restriction $0 \leq t \leq 3$

(b) Find the maximum distance of the yo-yo from the child's hand, correct to 3sf.

a) $s = 0.2t(3 + 2t - t^2) = 0.2t(3 - t)(1 + t)$

$t \geq 0$ as time cannot be negative.

If $t > 3, s < 0$ (but distance cannot be negative)



b) 1.21 m (3 sf)

Graphs used with permission from DESMOS: https://www.desmos.com/

Worked example	Your turn
A particle P is moving along the x-axis. At time t seconds, the velocity of P in the direction of x increasing, is: $$v = \frac{5}{3}t^3 - 18t^2 + 36t$$ Find the maximum velocity of the particle	A particle P is moving along the x-axis. At time t seconds, the velocity of P in the direction of x increasing, is: $$v = t^3 - 16t^2 + 64t$$ Find the maximum velocity of the particle $75.9\ ms^{-1}$ (3 sf)

Worked example

A particle P is moving along the x-axis. At time t seconds, the velocity of P in the direction of x increasing, is:

$$v = 3t^2 - 21t + 30, t \geq 0$$

Find the maximum speed of the particle

Your turn

A particle P is moving along the x-axis. At time t seconds, the velocity of P in the direction of x increasing, is:

$$v = 2t^2 - 14t + 20, t \geq 0$$

Find the maximum speed of the particle

$$20\ ms^{-1}$$

11.4) Using integration

Worked example	Your turn
A particle is moving on the x-axis. At time $t = 0$, the particle is at the point where $x = 7$. The velocity of the particle at time t seconds (where $t \geq 0$) is $(8t - 3t^2)$ ms^{-1}. Find: (a) An expression for the displacement of the particle from O at time t seconds. (b) The distance of the particle from its starting point when $t = 4$.	A particle is moving on the x-axis. At time $t = 0$, the particle is at the point where $x = 5$. The velocity of the particle at time t seconds (where $t \geq 0$) is $(6t - t^2)$ ms^{-1}. Find: (a) An expression for the displacement of the particle from O at time t seconds. (b) The distance of the particle from its starting point when $t = 6$. a) $x = 3t^2 - \frac{1}{3}t^3 + 5$ b) $36\ m$

Worked example	Your turn
A particle travels in a straight line. After t seconds its velocity, v ms^{-1}, is given by $v = 7 - 6t^2$, $t \geq 0$. Find the distance travelled by the particle in the fifth second of its motion.	A particle travels in a straight line. After t seconds its velocity, v ms^{-1}, is given by $v = 5 - 3t^2$, $t \geq 0$. Find the distance travelled by the particle in the third second of its motion. $14\ m$

Worked example	Your turn
A particle P moves on the positive x-axis. The velocity of P at time t seconds is $(4t^2 - 9t + 2)ms^{-1}$. When $t = 0, P$ is $5\ m$ from the origin O. Find: a) The values of t when P is instantaneously at rest b) The acceleration of P when $t = 10$ c) The total distance travelled by P in the interval $0 \leq t \leq 3$	A particle P moves on the positive x-axis. The velocity of P at time t seconds is $(2t^2 - 9t + 4)ms^{-1}$. When $t = 0, P$ is $15\ m$ from the origin O. Find: a) The values of t when P is instantaneously at rest b) The acceleration of P when $t = 5$ c) The total distance travelled by P in the interval $0 \leq t \leq 5$ a) $t = \frac{1}{2}, t = 4$ b) $11\ ms^{-2}$ c) $19.4\ m$ (3 sf)

Worked example	Your turn
A particle travels in a straight line such that its acceleration, $a\ ms^{-2}$, at time t seconds, is given by $a = 18t + 6$. When $t = 2$ seconds, the displacement, s, is 40 metres. When $t = 3$ seconds, the displacement is 117 metres. Find: a) The displacement when $t = 4$ seconds. b) The velocity when $t = 4$ seconds.	A particle travels in a straight line such that its acceleration, $a\ ms^{-2}$, at time t seconds, is given by $a = 12t + 4$. When $t = 1$ seconds, the displacement, s, is 6 metres. When $t = 2$ seconds, the displacement is 196 metres. Find: a) The displacement when $t = 3$ seconds. b) The velocity when $t = 3$ seconds. a) $98\ m$ b) $76\ ms^{-1}$

11.5) Constant acceleration formulae

Worked example	Your turn
A particle moves in a straight line with constant acceleration $a\ ms^{-2}$. Given that its initial velocity is $u\ ms^{-1}$ and its initial displacement is $0\ m$, prove that: Its velocity, $v\ ms^{-1}$, at time t s is given by $v = u + at$	A particle moves in a straight line with constant acceleration $a\ ms^{-2}$. Given that its initial velocity is $u\ ms^{-1}$ and its initial displacement is $0\ m$, prove that: Its displacement, $s\ m$, at time t s is given by $s = ut + \frac{1}{2}at^2$ Proof