# 10.4) Applications to modelling

| Worked example | Your turn |
| --- | --- |
| The price of a car in £s, $x$ years after purchase, is modelled by the function $f(x) = 5000\,(0.58)^x - 100 \sin x\,, \qquad x > 0$ (a) Use the model to find the value, to the nearest hundred £s, of the car 5 years after purchase. | The price of a car in £s, $x$ years after purchase, is modelled by the function $g(x) = 15000\,(0.85)^x - 1000 \sin x\,, \qquad x > 0$ (a) Use the model to find the value, to the nearest hundred £s, of the car 10 years after purchase. (a) £3500 |

# Worked example

The price of a car in £s, $x$ years after purchase, is modelled by the function

$$f(x) = 5000\,(0.58)^x - 100\sin x\,, \qquad x > 0$$

(a) Use the model to find the value, to the nearest hundred £s, of the car 5 years after purchase.

(b) Show that $f(x)$ has a root between 7 and 8.

# Your turn

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(a) Use the model to find the value, to the nearest hundred £s, of the car 10 years after purchase.

(b) Show that $g(x)$ has a root between 19 and 20.

(b)

$g(19) = 543.11 \ldots > 0$

$g(20) = -331.55 \ldots < 0$

Change of sign and $g(x)$ continuous in the interval $[19, 20]$

$\therefore$ root in the interval $[19, 20]$

| Worked example | Your turn |
|---|---|
| The price of a car in £s, $x$ years after purchase, is modelled by the function | The price of a car in £s, $x$ years after purchase, is modelled by the function |

## Worked example

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(a) Use the model to find the value, to the nearest hundred £s, of the car 5 years after purchase.

(b) Show that $f(x)$ has a root between 7 and 8.

(c) Taking 7.5 as a first approximation, apply the Newton-Raphson method once to $f(x)$ to obtain a second approximation for the time when the value of the car is zero. Give your answer to 3 decimal places.

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(c)

$g'(x) = (15\,000)(0.85)^x(\ln 0.85) - 1000\cos x$

$g(19.5) = 15000(0.85)^{19.5} - 1000\sin(19.5)$

$= 25.0693\ldots$

$g'(19.5) = 15000(0.85)^{19.5}(\ln 0.85) - 1000\cos(19.5)$

$= -893.3009\ldots$

$x_1 = 19.5 - \frac{g(19.5)}{g'(19.5)} = 19.528$ (3 dp)

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(c) Taking 7.5 as a first approximation, apply the Newton-Raphson method once to $f(x)$ to obtain a second approximation for the time when the value of the car is zero. Give your answer to 3 decimal places.

(d) Criticise this model with respect to the value of the car as it gets older.

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(c) Taking 19.5 as a first approximation, apply the Newton-Raphson method once to $g(x)$ to obtain a second approximation for the time when the value of the car is zero. Give your answer to 3 decimal places.

(d) Criticise this model with respect to the value of the car as it gets older.

(d) In reality, the car can never have a negative value so this model is not reasonable for cars that are approximately 20 or more years old.