10.2) Forces as vectors

Worked example

The forces $3\boldsymbol{i} - 2\boldsymbol{j}$, $-4\boldsymbol{i} + \boldsymbol{j}$, $-2\boldsymbol{i} - 3\boldsymbol{j}$ and $a\boldsymbol{i} + b\boldsymbol{j}$ act on an object which is in equilibrium. Find the values of a and b.

Your turn

The forces $2\boldsymbol{i} + 3\boldsymbol{j}$, $4\boldsymbol{i} - \boldsymbol{j}$, $-3\boldsymbol{i} + 2\boldsymbol{j}$ and $a\boldsymbol{i} + b\boldsymbol{j}$ act on an object which is in equilibrium. Find the values of a and b.

$$a = -3, b = -4$$

Worked example

The vector i is due east and j due north.
A particle begins at rest at the origin.
It is acted on by three forces $(3\boldsymbol{i} - \boldsymbol{j})$ N, $(2\boldsymbol{i} + 3\boldsymbol{j})$ N and $(-4\boldsymbol{i} + \boldsymbol{j})$ N.

(a) Find the resultant force in the form $p\boldsymbol{i} + q\boldsymbol{j}$.
(b) Work out the magnitude and bearing of the resultant force.

Your turn

The vector i is due east and j due north.
A particle begins at rest at the origin.
It is acted on by three forces $(2\boldsymbol{i} + \boldsymbol{j})$ N, $(3\boldsymbol{i} - 2\boldsymbol{j})$ N and $(-\boldsymbol{i} + 4\boldsymbol{j})$ N.

(a) Find the resultant force in the form $p\boldsymbol{i} + q\boldsymbol{j}$.
(b) Work out the magnitude and bearing of the resultant force.

a) $4\boldsymbol{i} + 3\boldsymbol{j}$
b) $053.1°$ (1 dp)

Worked example

Three forces F_1, F_2 and F_3 acting on a particle P are:

$$F_1 = (9\boldsymbol{i} - 7\boldsymbol{j})\ N$$
$$F_2 = (6\boldsymbol{i} + 5\boldsymbol{j})\ N$$
$$F_3 = (p\boldsymbol{i} + q\boldsymbol{j})\ N$$

where p and q are constants.

Given that P is in equilibrium,

a) Find the value of p and the value of q

The force F_3 is now removed. The resultant of F_1 and F_2 is R. Find:

b) The magnitude of R

c) The angle, to the nearest degree, that the direction of R makes with $\boldsymbol{j}$.

Your turn

Three forces F_1, F_2 and F_3 acting on a particle P are:

$$F_1 = (7\boldsymbol{i} - 9\boldsymbol{j})\ N$$
$$F_2 = (5\boldsymbol{i} + 6\boldsymbol{j})\ N$$
$$F_3 = (p\boldsymbol{i} + q\boldsymbol{j})\ N$$

where p and q are constants.

Given that P is in equilibrium,

a) Find the value of p and the value of q

The force F_3 is now removed. The resultant of F_1 and F_2 is R. Find:

b) The magnitude of R

c) The angle, to the nearest degree, that the direction of R makes with $\boldsymbol{j}$.

a) $p = -12, q = 3$

b) $12.4\ N$ (3 sf)

c) $104°$

Worked example

Two forces F_1 and F_2 acting on a particle P are:

$$F_1 = (3\boldsymbol{i} - 2\boldsymbol{j})\ N$$
$$F_2 = (p\boldsymbol{i} + 3p\boldsymbol{j})\ N$$

where p is a positive constant.

a) Find the angle between F_2 and $\boldsymbol{i}$

The resultant of F_1 and F_2 is R.

b) Given that R is parallel to $\boldsymbol{j}$, find the value of p

Your turn

Two forces F_1 and F_2 acting on a particle P are:

$$F_1 = (\boldsymbol{i} - 3\boldsymbol{j})\ N$$
$$F_2 = (p\boldsymbol{i} + 2p\boldsymbol{j})\ N$$

where p is a positive constant.

a) Find the angle between F_2 and $\boldsymbol{j}$

The resultant of F_1 and F_2 is R.

b) Given that R is parallel to $\boldsymbol{i}$, find the value of p

a) $26.6°$

b) $p = \frac{3}{2}$

Worked example

Two forces F_1 and F_2 acting on a particle P are:

$$F_1 = (3\boldsymbol{i} - 2\boldsymbol{j})\ N$$
$$F_2 = (p\boldsymbol{i} + 3p\boldsymbol{j})\ N$$

where p is a positive constant.

The resultant of F_1 and F_2 is R.

Given that R is parallel to $13\boldsymbol{i} + 10\boldsymbol{j}$, find the value of p

Your turn

Two forces F_1 and F_2 acting on a particle P are:

$$F_1 = (2\boldsymbol{i} - 3\boldsymbol{j})\ N$$
$$F_2 = (p\boldsymbol{i} + 2p\boldsymbol{j})\ N$$

where p is a positive constant.

The resultant of F_1 and F_2 is R.

Given that R is parallel to $12\boldsymbol{i} + 11\boldsymbol{j}$, find the value of p

$$p = \frac{58}{13}$$