

## 10.1) Locating roots

## Worked example

Show that  $f(x) = e^x + 3x - 2$  has a root between  $x = 0.2$  and  $x = 0.3$

## Your turn

Show that  $f(x) = e^x + 2x - 3$  has a root between  $x = 0.5$  and  $x = 0.6$

$$f(0.5) = -0.351 \dots < 0$$

$$f(0.6) = 0.022 \dots > 0$$

Change of sign and  $f(x)$  continuous in the interval  $[0.5, 0.6]$

$\therefore$  Root in the interval  $[0.5, 0.6]$

## Worked example

Explain why there are no real roots to

$$f(x) = \frac{1}{x-2} \text{ between } x = 1 \text{ and } x = 3$$

## Your turn

Explain why there are no real roots to

$$f(x) = \frac{1}{x} \text{ between } x = -1 \text{ and } x = 1$$

$f(x)$  not continuous in the interval  
[−1, 1]

## Worked example

Using the same axes, sketch the graphs of

$$y = e^x \text{ and } y = \frac{1}{x}$$

a) Explain how your diagram shows that the function  $f(x) = e^x - \frac{1}{x}$  has only one root

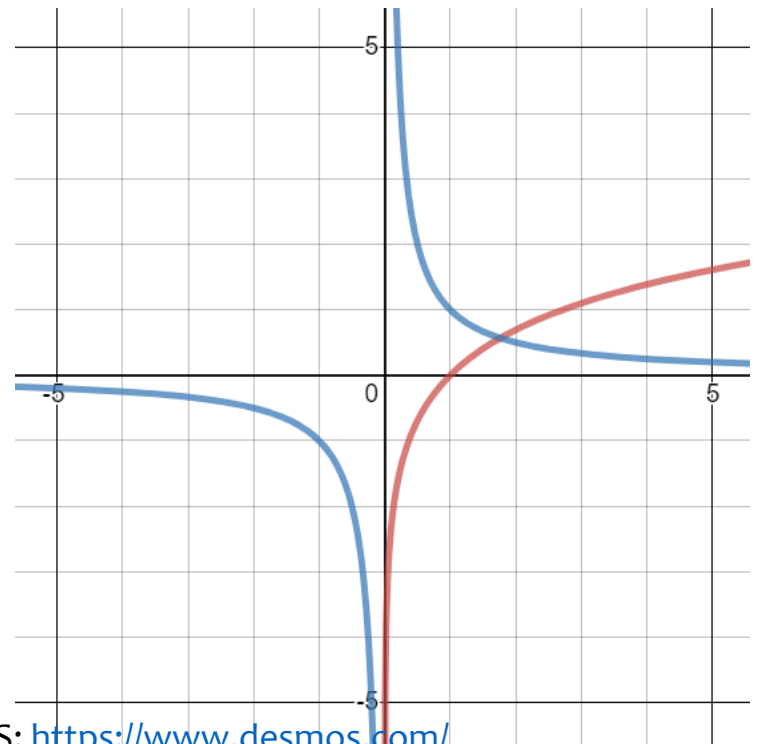
## Your turn

Using the same axes, sketch the graphs of

$$y = \ln x \text{ and } y = \frac{1}{x}$$

a) Explain how your diagram shows that the function  $f(x) = \ln x - \frac{1}{x}$  has only one root

a) The lines intersect where  $\ln x = \frac{1}{x} \rightarrow \ln x - \frac{1}{x} = 0$   
Thus the roots of  $f(x)$  correspond to the points of intersection, and there is only one point of intersection on the graph.



## Worked example

Using the same axes, sketch the graphs of

$$y = e^x \text{ and } y = \frac{1}{x}$$

- a) Explain how your diagram shows that the function  $f(x) = e^x - \frac{1}{x}$  has only one root
- b) Show that this root lies in the interval  $0.5 < x < 0.6$

## Your turn

Using the same axes, sketch the graphs of

$$y = \ln x \text{ and } y = \frac{1}{x}$$

- a) Explain how your diagram shows that the function  $f(x) = \ln x - \frac{1}{x}$  has only one root
- b) Show that this root lies in the interval  $1.7 < x < 1.8$

$$\text{b) } f(1.7) = -0.0576 \dots < 0$$

$$f(1.8) = 0.0322 \dots > 0$$

Change of sign and  $f(x)$  continuous in the interval  $[1.7, 1.8]$

$\therefore$  Root in the interval  $[1.7, 1.8]$

## Worked example

Using the same axes, sketch the graphs of

$$y = e^x \text{ and } y = \frac{1}{x}$$

- Explain how your diagram shows that the function  $f(x) = e^x - \frac{1}{x}$  has only one root
- Show that this root lies in the interval  $0.5 < x < 0.6$
- Show that the root is 0.567 to 3 decimal places

## Your turn

Using the same axes, sketch the graphs of

$$y = \ln x \text{ and } y = \frac{1}{x}$$

- Explain how your diagram shows that the function  $f(x) = \ln x - \frac{1}{x}$  has only one root
- Show that this root lies in the interval  $1.7 < x < 1.8$
- Show that the root is 1.763 to 3 decimal places

$$c) \quad f(1.7625) = -0.00064 < 0$$

$$f(1.7635) = 0.00024 > 0$$

Change of sign and  $f(x)$  continuous in the interval  $[1.7625, 1.7635]$

$$\therefore 1.7625 < \alpha < 1.7635,$$

$$\therefore \alpha = 1.763 \text{ correct to 3dp.}$$