10) Trigonometric identities and equations

10.1) Angles in all four quadrants

Worked example	Your turn
Given that θ is an acute angle, express in terms of θ: • $\sin(-\theta)$ • $\cos(-\theta)$	Given that θ is an acute angle, express in terms of θ: • $\tan(-\theta)$ $-\tan\theta$

10.2) Exact values of trigonometrical ratios

Worked example	Your turn
Without a calculator, work out the value of:	Without a calculator, work out the value of:
• $\tan(315°)$	• $\tan(225°)$ 1
• $\tan(-120°)$	• $\tan(210°)$ $\dfrac{1}{\sqrt{3}}$
• $\sin(330°)$	• $\sin(150°)$ $\dfrac{1}{2}$
• $\cos(240°)$	• $\cos(300°)$ $\dfrac{1}{2}$
• $\sin(-135°)$	• $\sin(-45°)$ $-\dfrac{1}{\sqrt{2}}$
• $\cos(675°)$	• $\cos(750°)$ $\dfrac{\sqrt{3}}{2}$
• $\cos(150°)$	• $\cos(120°)$ $-\dfrac{1}{2}$

10.3) Trigonometric identities

Worked example	Your turn
Simplify:	Simplify:
$$\sin^2 \frac{1}{3}y + \cos^2 \frac{1}{3}y$$	$$\sin^2 3x + \cos^2 3x$$
	1
$$\cos^2(3\theta - 10) + \sin^2(3\theta - 10)$$	

Worked example	Your turn
Prove that $1 - \dfrac{\tan\theta\cos^3\theta}{\sin\theta} \equiv \sin^2\theta$	Prove that $1 - \tan\theta\sin\theta\cos\theta \equiv \cos^2\theta$
	Proof

Worked example	Your turn
Prove that $\tan \theta - \frac{1}{\tan \theta} \equiv \frac{1 - 2\cos^2 \theta}{\sin \theta \cos \theta}$	Prove that $\tan \theta + \frac{1}{\tan \theta} \equiv \frac{1}{\sin \theta \cos \theta}$ Proof

Worked example	Your turn
Simplify $10 - 10\cos^2\theta$	Simplify $5 - 5\sin^2\theta$
	$5\cos^2\theta$

Worked example	Your turn
Simplify: $$\frac{\cos 4\theta}{\sqrt{1 - \sin^2 4\theta}}$$	Simplify: $$\frac{\sin 2\theta}{\sqrt{1 - \sin^2 2\theta}}$$ $$\tan 2\theta$$

Worked example	Your turn
Prove that $$\frac{\sin^4\theta - \cos^4\theta}{\sin^2\theta} \equiv 1 - \frac{1}{\tan^2\theta}$$	Prove that $$\frac{\cos^4\theta - \sin^4\theta}{\cos^2\theta} \equiv 1 - \tan^2\theta$$ Proof

Worked example	Your turn
Prove that $$\frac{\dfrac{\sin x}{\tan x}}{\sqrt{1 - \sin^2 x}} \equiv 1$$	Prove that $$\frac{\tan x \cos x}{\sqrt{1 - \cos^2 x}} \equiv 1$$ Proof

Worked example	Your turn
Prove that $$\frac{1}{\tan^2 \theta} \equiv \frac{1}{\sin^2 \theta} - 1$$	Prove that $$\tan^2 \theta \equiv \frac{1}{\cos^2 \theta} - 1$$ Proof

Worked example	Your turn
Given that $\sin\theta = \frac{3}{7}$ and that θ is acute, find the exact value of $\cos\theta$	Given that $\sin\theta = \frac{2}{5}$ and that θ is obtuse, find the exact value of $\cos\theta$
	$$-\frac{\sqrt{21}}{5}$$

Worked example	Your turn
Given that $\cos\theta = -\dfrac{5}{13}$ and that θ is obtuse, find the value of $\sin\theta$	Given that $\cos\theta = -\dfrac{3}{5}$ and that θ is reflex, find the value of $\sin\theta$
	$$-\frac{4}{5}$$

Worked example	Your turn
Given that $\tan \theta = -\dfrac{5}{12}$ and that θ is reflex, find the value of $\sin \theta$ and $\cos \theta$	Given that $\tan \theta = \dfrac{3}{4}$ and that θ is acute, find the value of $\sin \theta$ and $\cos \theta$

$$\sin \theta = \frac{3}{5}$$
$$\cos \theta = \frac{4}{5}$$

Worked example	Your turn
Given that $p = 4\cos\theta$ and $q = 5\sin\theta$, show that $25p^2 + 16q^2 = 400$	Given that $p = 3\cos\theta$ and $q = 2\sin\theta$, show that $4p^2 + 9q^2 = 36$ Shown

10.4) Simple trigonometric equations

Worked example	Your turn
Solve in the interval $0 \leq \theta \leq 360°$: $$\sin \theta = \frac{\sqrt{3}}{2}$$	Solve in the interval $0 \leq \theta \leq 360°$: $$\sin \theta = \frac{1}{2}$$ $$\theta = 30°, 150°$$
$$\sin \theta = -\frac{\sqrt{2}}{2}$$	

Worked example	Your turn
Solve in the interval $0 \leq \theta \leq 360°$:	Solve in the interval $0 \leq \theta \leq 360°$:
$$\cos \theta = \frac{\sqrt{3}}{2}$$	$$\cos \theta = \frac{1}{2}$$
	$\theta = 60°, 300°$
$$\cos \theta = -\frac{\sqrt{2}}{2}$$	

Worked example	Your turn
Solve in the interval $0 \leq \theta \leq 360°$: $$\tan \theta = \frac{\sqrt{3}}{3}$$	Solve in the interval $0 \leq \theta \leq 360°$: $$\tan \theta = \sqrt{3}$$ $$\theta = 60°, 240°$$
$$\tan \theta = -1$$	

Worked example	Your turn
Solve in the interval $0 \leq \theta \leq 360°$: $$3 \cos \theta = -4$$	Solve in the interval $0 \leq \theta \leq 360°$: $$5 \sin \theta = -2$$ $\theta = 204°, 336°$ (3 sf)
$$5 \tan \theta = 7$$	

Worked example	Your turn
Solve in the interval $0 \leq \theta \leq 360°$: $$\sqrt{3}\ \sin\theta = \cos\theta$$	Solve in the interval $0 \leq \theta \leq 360°$: $$\sin\theta = \sqrt{3}\cos\theta$$ $$\theta = 60°, 240°$$

Worked example	Your turn
Solve in the interval $0 \le \theta \le 360°$: $$2\sin\theta + 3\cos\theta = 0$$	Solve in the interval $0 \le \theta \le 360°$: $$5\sin\theta - 4\cos\theta = 0$$ $\theta = 38.7°, 218.7° \,(1\,dp)$

Worked example	Your turn
Solve in the interval $0 \leq x \leq 360°$: $$5\cos^2 x - 3\sin^2 x = 4$$	Solve in the interval $0 \leq x \leq 360°$: $$8\sin^2 x - 7\cos^2 x = 6$$ $x = 84.6°, 95.4°, 256.0°, 264.6°$ (1 dp)

Worked example	Your turn
Explain why there are no solutions to the equation $3\sin^2 x - 4\cos^2 x = 5$	Explain why there are no solutions to the equation $3\sin^2 x + 4\cos^2 x = 5$

$$\sin^2 x = -1$$

No real solutions

Or

$$\cos^2 x = 2$$

$$\cos x = \pm\sqrt{2} \text{ but } -1 \leq \cos x \leq 1$$

$\therefore$ No solutions

Worked example	Your turn
Solve in the interval $0 \leq x \leq 360°$: $$\sin 3x = \frac{1}{2}$$	Solve in the interval $0 \leq x \leq 360°$: $$\cos 3x = -\frac{1}{2}$$ $x = 40°, 80°, 160°, 200°, 280°, 320°$
$$\tan 4x = -\sqrt{3}$$	

Worked example	Your turn
Solve in the interval $0 \leq x \leq 360°$: $$4 \sin 3x = 5 \cos 3x$$	Solve in the interval $0 \leq x \leq 360°$: $$2 \sin 2x = \cos 2x$$ $x = 13.3°, 103.3°, 193.3°, 283.3°$ (1 dp)

Worked example	Your turn
Solve in the interval $0 \leq x \leq 360°$: $$\cos(x + 30°) = 0.6$$	Solve in the interval $0 \leq x \leq 360°$: $$\sin(x + 60°) = 0.3$$ $x = 102.5°, 317.5°\ (\text{1 dp})$

Worked example	Your turn
Solve in the interval $0 \leq x \leq 360°$: $$\sin(4x + 60°) = \frac{1}{2}$$	Solve in the interval $0 \leq x \leq 360°$: $$\sin(2x + 30°) = \frac{\sqrt{2}}{2}$$ $x = 7.5°, 52.5°, 187.5°, 232.5°$ (1 dp)

Worked example	Your turn
Solve in the interval $0 \leq x < 180°$: $$\cos(2x - 50°) = -0.3$$	Solve in the interval $0 \leq x < 180°$: $$\cos(3x - 10°) = -0.4$$ $x = 41.2°, 85.5°, 161.2°$ (1 dp)

10.6) Equations and identities

Worked example	Your turn
Solve in the interval $0 \leq x < 360°$: $$3 \sin^2 x - 5 \sin x + 2 = 0$$	Solve in the interval $0 \leq x < 360°$: $$5 \sin^2 x + 3 \sin x - 2 = 0$$ $x = 23.6°, 156.4°, 270.0°$ (1 dp)

Worked example	Your turn
Solve in the interval $0 \leq x < 360°$: $$5\cos^2 x + 3\cos x - 2 = 0$$	Solve in the interval $0 \leq x < 360°$: $$3\cos^2 x - 5\cos x + 2 = 0$$ $x = 0.0°, 48.2°, 180.0°, 311.8°$ (1 dp)

Worked example	Your turn
Solve in the interval $0 \leq x < 360°$: $3\sin^2 x - 7\sin x + 4 = 0$	Solve in the interval $0 \leq x < 360°$: $2\cos^2 x - \cos x - 3 = 0$ $x = 180°$

Worked example	Your turn
Solve in the interval $0 \leq x < 360°$: $$\sin^2 x = 2 \sin x$$	Solve in the interval $0 \leq x \leq 360°$: $$\cos^2 x = 4 \cos x$$ $$x = 0°$$

Worked example	Your turn
Solve in the interval $0 \leq x \leq 360°$: $$\cos^2 x - \cos x = 0$$	Solve in the interval $0 \leq x < 360°$: $$\sin^2 x + 3 \sin x = 0$$ $x = 0°, 180°$

Worked example	Your turn
Solve in the interval $0 \leq x \leq 360°$: $$\cos^2(x - 60°) = \frac{\sqrt{3}}{2}$$	Solve in the interval $0 \leq x \leq 360°$: $$\sin^2(x - 30°) = \frac{1}{2}$$ $$x = 75°, 165°, 255°, 345°$$

Worked example	Your turn
Solve in the interval $0 \leq \theta \leq 360°$: $$\tan^2 \theta = 25$$	Solve in the interval $0 \leq \theta \leq 360°$: $$\tan^2 \theta = 16$$ $\theta = 76.0°, 104.0°, 256.0°, 284.0°$ (1 dp)

Worked example	Your turn
Solve in the interval $0 \leq x \leq 360°$: $$5\cos^2 x - 4\sin x + 3 = 0$$	Solve in the interval $0 \leq x \leq 360°$: $$6\sin^2 x + 7\cos x - 2 = 0$$ $x = 114.8°, 245.2°$ (1 dp)

Worked example	Your turn
Solve in the interval $-180° \leq x \leq 180$: $2\sin^2 x - 9\cos x = 3\cos^2 x$	Solve in the interval $-180° \leq x \leq 180$: $2\cos^2 x + 9\sin x = 3\sin^2 x$ $x = -168.5°, -11.5°$ (1 dp)