10) Forces and motion

10.1) Force diagrams

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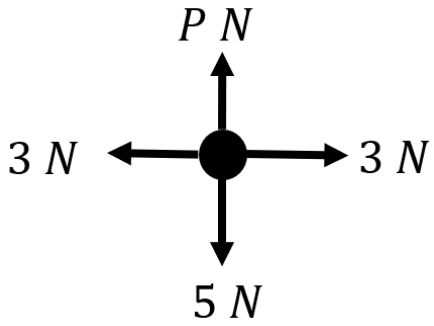
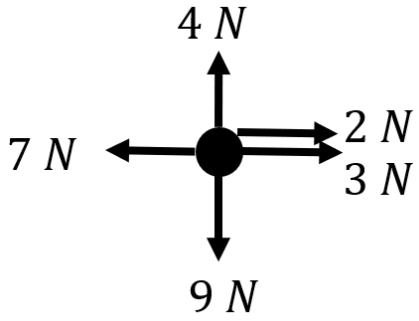
Worked example

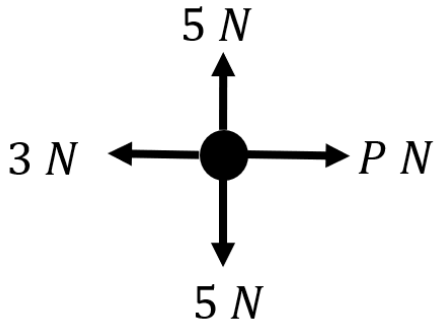
Draw a force diagram to represent the resultant force:





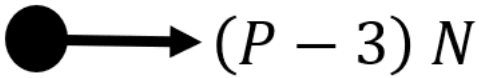
Your turn

Draw a force diagram to represent the resultant force:



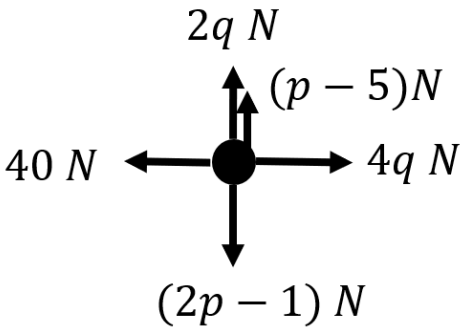
$R(\uparrow): 0\ N$

$R(\rightarrow): (P-3)N$



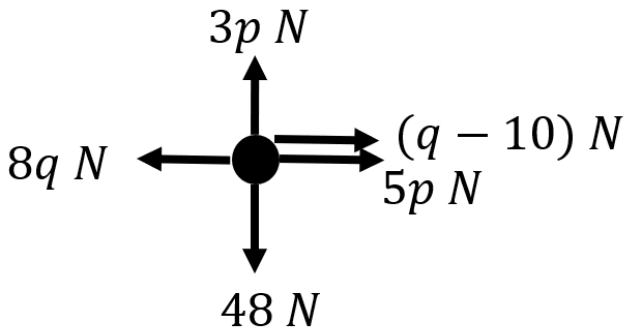
Worked example

A particle is acted on a by a set of forces.
Given that the particle is at rest, find the values of p and q



Your turn

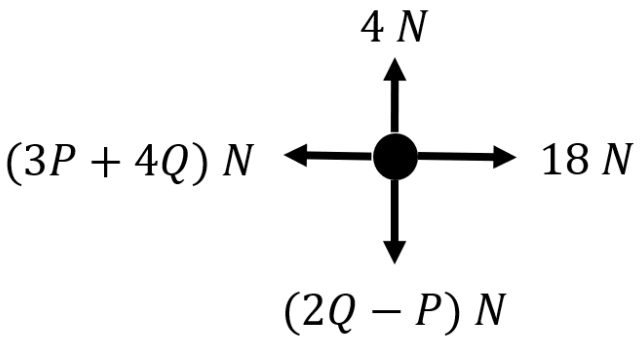
A particle is acted on a by a set of forces.
Given that the particle is at rest, find the values of p and q



$p = 16, q = 10$

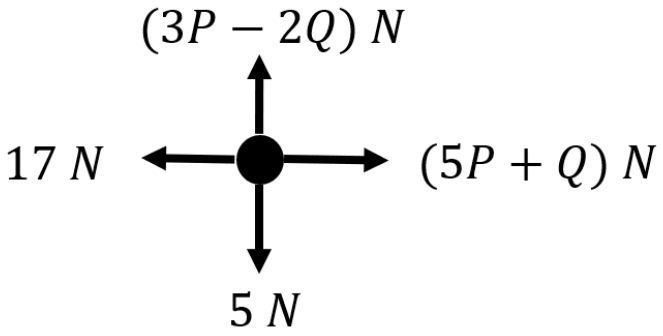
Worked example

Given that the particle is moving with constant velocity, find the values of P and Q



Your turn

Given that the particle is moving with constant velocity, find the values of P and Q



$P = 3, Q = 2$

10.2) Forces as vectors

Chapter CONTENTS

Worked example

The forces $3\boldsymbol{i} - 2\boldsymbol{j}$, $-4\boldsymbol{i} + \boldsymbol{j}$, $-2\boldsymbol{i} - 3\boldsymbol{j}$ and $a\boldsymbol{i} + b\boldsymbol{j}$ act on an object which is in equilibrium. Find the values of a and b.

Your turn

The forces $2\boldsymbol{i} + 3\boldsymbol{j}$, $4\boldsymbol{i} - \boldsymbol{j}$, $-3\boldsymbol{i} + 2\boldsymbol{j}$ and $a\boldsymbol{i} + b\boldsymbol{j}$ act on an object which is in equilibrium. Find the values of a and b.

$$a = -3, b = -4$$

Worked example

The vector i is due east and j due north.
A particle begins at rest at the origin.
It is acted on by three forces $(3\boldsymbol{i} - \boldsymbol{j})$ N, $(2\boldsymbol{i} + 3\boldsymbol{j})$ N and $(-4\boldsymbol{i} + \boldsymbol{j})$ N.

(a) Find the resultant force in the form $p\boldsymbol{i} + q\boldsymbol{j}$.
(b) Work out the magnitude and bearing of the resultant force.

Your turn

The vector i is due east and j due north.
A particle begins at rest at the origin.
It is acted on by three forces $(2\boldsymbol{i} + \boldsymbol{j})$ N, $(3\boldsymbol{i} - 2\boldsymbol{j})$ N and $(-\boldsymbol{i} + 4\boldsymbol{j})$ N.

(a) Find the resultant force in the form $p\boldsymbol{i} + q\boldsymbol{j}$.
(b) Work out the magnitude and bearing of the resultant force.

a) $4\boldsymbol{i} + 3\boldsymbol{j}$
b) $053.1°$ (1 dp)

Worked example

Three forces F_1, F_2 and F_3 acting on a particle P are:

$$F_1 = (9\boldsymbol{i} - 7\boldsymbol{j})\ N$$
$$F_2 = (6\boldsymbol{i} + 5\boldsymbol{j})\ N$$
$$F_3 = (p\boldsymbol{i} + q\boldsymbol{j})\ N$$

where p and q are constants.

Given that P is in equilibrium,

a) Find the value of p and the value of q

The force F_3 is now removed. The resultant of F_1 and F_2 is R. Find:

b) The magnitude of R

c) The angle, to the nearest degree, that the direction of R makes with $\boldsymbol{j}$.

Your turn

Three forces F_1, F_2 and F_3 acting on a particle P are:

$$F_1 = (7\boldsymbol{i} - 9\boldsymbol{j})\ N$$
$$F_2 = (5\boldsymbol{i} + 6\boldsymbol{j})\ N$$
$$F_3 = (p\boldsymbol{i} + q\boldsymbol{j})\ N$$

where p and q are constants.

Given that P is in equilibrium,

a) Find the value of p and the value of q

The force F_3 is now removed. The resultant of F_1 and F_2 is R. Find:

b) The magnitude of R

c) The angle, to the nearest degree, that the direction of R makes with $\boldsymbol{j}$.

a) $p = -12, q = 3$

b) $12.4\ N$ (3 sf)

c) $104°$

Worked example

Two forces F_1 and F_2 acting on a particle P are:

$$F_1 = (3\boldsymbol{i} - 2\boldsymbol{j})\ N$$
$$F_2 = (p\boldsymbol{i} + 3p\boldsymbol{j})\ N$$

where p is a positive constant.

a) Find the angle between F_2 and $\boldsymbol{i}$

The resultant of F_1 and F_2 is R.

b) Given that R is parallel to $\boldsymbol{j}$, find the value of p

Your turn

Two forces F_1 and F_2 acting on a particle P are:

$$F_1 = (\boldsymbol{i} - 3\boldsymbol{j})\ N$$
$$F_2 = (p\boldsymbol{i} + 2p\boldsymbol{j})\ N$$

where p is a positive constant.

a) Find the angle between F_2 and $\boldsymbol{j}$

The resultant of F_1 and F_2 is R.

b) Given that R is parallel to $\boldsymbol{i}$, find the value of p

a) $26.6°$

b) $p = \frac{3}{2}$

Worked example

Two forces F_1 and F_2 acting on a particle P are:

$$F_1 = (3\boldsymbol{i} - 2\boldsymbol{j})\ N$$
$$F_2 = (p\boldsymbol{i} + 3p\boldsymbol{j})\ N$$

where p is a positive constant.

The resultant of F_1 and F_2 is R.

Given that R is parallel to $13\boldsymbol{i} + 10\boldsymbol{j}$, find the value of p

Your turn

Two forces F_1 and F_2 acting on a particle P are:

$$F_1 = (2\boldsymbol{i} - 3\boldsymbol{j})\ N$$
$$F_2 = (p\boldsymbol{i} + 2p\boldsymbol{j})\ N$$

where p is a positive constant.

The resultant of F_1 and F_2 is R.

Given that R is parallel to $12\boldsymbol{i} + 11\boldsymbol{j}$, find the value of p

$$p = \frac{58}{13}$$

10.3) Forces and acceleration

Chapter CONTENTS

Worked example

A car of 1000kg has a driving force of 1600N and forces of 400N resisting its motion.
Determine its acceleration.

Your turn

A car of 2000kg has a driving force of 800N and forces of 200N resisting its motion.
Determine its acceleration.

$0.3\ ms^{-2}$

Worked example

An object of mass 140kg experiences air resistance of 600 N. Determine the object's acceleration as it falls towards the ground.

Your turn

An object of mass 70kg experiences air resistance of 300 N. Determine the object's acceleration as it falls towards the ground.

$5.51\ ms^{-2}$ (3 sf)

Worked example

An adult has a mass of 100kg. What is the gravitational force (weight) acting on the adult?

Your turn

A child has a mass of 50kg. What is the gravitational force (weight) acting on the child?

$490\ N$

Worked example

A body of mass 10kg is pulled along a rough horizontal table by a horizontal force of magnitude 40N against a constant friction force of magnitude 8N. Given that the body is initially at rest, find:

(a) the acceleration of the body

(b) the distance travelled by the body in the first 2 seconds

(c) the magnitude of the normal reaction between the body and the table

Your turn

A body of mass 5kg is pulled along a rough horizontal table by a horizontal force of magnitude 20N against a constant friction force of magnitude 4N. Given that the body is initially at rest, find:

(a) the acceleration of the body

(b) the distance travelled by the body in the first 4 seconds

(c) the magnitude of the normal reaction between the body and the table

a) $3.2\ ms^{-2}$

b) $25.6\ m$

c) $49\ N$

Worked example

An objects of mass $8\ kg$ hits soft ground at a speed of $14\ ms^{-1}$ and sinks vertically downwards before coming to rest. The ground is assumed to exert a constant resistive force of magnitude $5000\ N$.

Find the vertical distance that the object sinks into the ground before coming to rest.

Your turn

An objects of mass $4\ kg$ hits soft ground at a speed of $28\ ms^{-1}$ and sinks vertically downwards before coming to rest. The ground is assumed to exert a constant resistive force of magnitude $5000\ N$.

Find the vertical distance that the object sinks into the ground before coming to rest.

$0.32\ m$ (2 sf)

Worked example

A lift of mass $500\ kg$ is lowered or raised by a metal cable attached to its top. The lift contains passengers whose total mass is $100\ kg$. The lift starts from rest and accelerates at a constant rate, reaching a speed of $5\ ms^{-1}$ after moving a distance of $4\ m$. Find:

a) The acceleration of the lift
b) The tension in the cable if the lift is moving vertically downwards
c) The tension in the cable if the lift is moving vertically upwards

Your turn

A lift of mass $400\ kg$ is lowered or raised by a metal cable attached to its top. The lift contains passengers whose total mass is $200\ kg$. The lift starts from rest and accelerates at a constant rate, reaching a speed of $4\ ms^{-1}$ after moving a distance of $5\ m$. Find:

a) The acceleration of the lift
b) The tension in the cable if the lift is moving vertically downwards
c) The tension in the cable if the lift is moving vertically upwards

a) $1.6\ ms^{-2}$
b) $4920\ N$
c) $6840\ N$

10.4) Motion in 2 dimensions

Chapter CONTENTS

Worked example

Let $\boldsymbol{i}$ represent East and $\boldsymbol{j}$ North. A resultant force of $(2\boldsymbol{i} + 7\boldsymbol{j})\ N$ acts upon a particle of mass 0.25 kg.

(a) Find the acceleration of the particle in the form $(p\boldsymbol{i} + q\boldsymbol{j})$ ms^{-2}.

(b) Find the magnitude and bearing of the acceleration of the particle.

Your turn

Let $\boldsymbol{i}$ represent East and $\boldsymbol{j}$ North. A resultant force of $(3\boldsymbol{i} + 8\boldsymbol{j})\ N$ acts upon a particle of mass 0.5 kg.

(a) Find the acceleration of the particle in the form $(p\boldsymbol{i} + q\boldsymbol{j})\ ms^{-2}$.

(b) Find the magnitude and bearing of the acceleration of the particle.

a) $(6\boldsymbol{i} + 16\boldsymbol{j})\ ms^{-2}$

b) Magnitude $= 17.1\ ms^{-2}$ (3 sf)

Bearing $= 020.6°$ (1 dp)

Worked example

A boat is modelled as a particle of mass 30 kg being acted on by three forces.

$$F_1 = \begin{pmatrix} 25 \\ 40 \end{pmatrix} N,$$

$$F_2 = \begin{pmatrix} 5q \\ 10q \end{pmatrix} N,$$

$$F_3 = \begin{pmatrix} 50 \\ -37.5 \end{pmatrix} N$$

Given that the boat is accelerating at a rate of $\begin{pmatrix} -0.75 \\ 0.4 \end{pmatrix}$ ms^{-2}, find the values of p and q.

Your turn

A boat is modelled as a particle of mass 60 kg being acted on by three forces.

$$F_1 = \begin{pmatrix} 80 \\ 50 \end{pmatrix} N,$$

$$F_2 = \begin{pmatrix} 10q \\ 20q \end{pmatrix} N,$$

$$F_3 = \begin{pmatrix} -75 \\ 100 \end{pmatrix} N$$

Given that the boat is accelerating at a rate of $\begin{pmatrix} 0.8 \\ -1.5 \end{pmatrix}$ ms^{-2}, find the values of p and q.

$$p = 4.3\,, q = -12$$

Worked example

A particle of mass $5\ kg$ start from rest and is acted upon by a force R of $(4\boldsymbol{i} + k\boldsymbol{j})\ N$. R acts on a bearing of $45°$. Find the value of k

Your turn

A particle of mass $4\ kg$ start from rest and is acted upon by a force R of $(5\boldsymbol{i} + k\boldsymbol{j})\ N$. R acts on a bearing of $135°$. Find the value of k

$$k = -5$$

Worked example

Two forces, $\begin{pmatrix}5\\2\end{pmatrix}$ N and $\begin{pmatrix}p\\q\end{pmatrix}$ N act on a particle of mass m kg. The resultant of the two forces is R.

a) Given that R acts in a direction which is parallel to the vector $\begin{pmatrix}-1\\2\end{pmatrix}$, show that $2p + q + 12 = 0$

b) Given also that $p = 1$ and that P moves with an acceleration of magnitude $10\sqrt{5}\ ms^{-2}$, find the value of m

Your turn

Two forces, $\begin{pmatrix}3\\4\end{pmatrix}$ N and $\begin{pmatrix}p\\q\end{pmatrix}$ N act on a particle of mass m kg. The resultant of the two forces is R.

a) Given that R acts in a direction which is parallel to the vector $\begin{pmatrix}-2\\1\end{pmatrix}$, show that $2q + p + 11 = 0$

b) Given also that $p = 5$ and that P moves with an acceleration of magnitude $40\sqrt{5}\ ms^{-2}$, find the value of m

a) Shown

b) $m = 0.1\ kg$

10.5) Connected particles

Chapter CONTENTS

Worked example

Two particles, P and Q, of masses 6kg and 4kg respectively, are connected by a light inextensible string. Particle Q is pulled by a horizontal force of magnitude 20N along a rough horizontal plane. Particle Q experiences a frictional force of 5N and particle P experiences a frictional force of 3N.

(a) Find the acceleration of the particles.

(b) Find the tension in the string.

(c) Explain how the modelling assumptions that the string is light and inextensible have been used.

Your turn

Two particles, P and Q, of masses 5kg and 3kg respectively, are connected by a light inextensible string. Particle P is pulled by a horizontal force of magnitude 40N along a rough horizontal plane. Particle P experiences a frictional force of 10N and particle Q experiences a frictional force of 6N.

(a) Find the acceleration of the particles.

(b) Find the tension in the string.

a) $a = 3\ ms^{-2}$

b) $T = 15\ N$

Worked example

A light scale-pan is attached to a vertical light inextensible string.
The scale-pan carries two masses A and B.
The mass of A is 300g and the mass of B is 200g.
A rests on top of B.
The scale-pan is raised vertically, using the string, with acceleration 0.25 ms^{-2}.

(a) Find the tension in the string.
(b) Find the force exerted on mass B by mass A.
(c) Find the force exerted on mass B by the scale-pan.

Your turn

A light scale-pan is attached to a vertical light inextensible string.
The scale-pan carries two masses A and B.
The mass of A is 400g and the mass of B is 600g.
A rests on top of B.
The scale-pan is raised vertically, using the string, with acceleration 0.5 ms^{-2}.

(a) Find the tension in the string.
(b) Find the force exerted on mass B by mass A.
(c) Find the force exerted on mass B by the scale-pan.

a) $10\ N$ (2 sf)
b) $4.1\ N$ (2 sf)
c) $10\ N$ (2 sf)

Worked example

A person travels in a lift. The mass of the person is $40\ kg$ and the mass of the lift is $860\ kg$.
The lift is being raised vertically by a vertical cable which is attached to the top of the lift. The lift is moving upwards and has constant deceleration $4\ ms^{-2}$. By modelling the cable as being light and inextensible, find:

a) The tension in the cable
b) The magnitude of the force exerted on the woman by the floor of the lift

Your turn

A person travels in a lift. The mass of the person is $50\ kg$ and the mass of the lift is $950\ kg$.
The lift is being raised vertically by a vertical cable which is attached to the top of the lift. The lift is moving upwards and has constant deceleration $2\ ms^{-2}$. By modelling the cable as being light and inextensible, find:

a) The tension in the cable
b) The magnitude of the force exerted on the woman by the floor of the lift

a) $7800\ N$
b) $390\ N$

Worked example

A car of mass $1200\ kg$ pulls a trailer of mass $400\ kg$ along a straight horizontal road using a light tow-bar which is parallel to the road. The horizontal resistances to motion of the car and the trailer have magnitudes $400\ N$ and $200\ N$ respectively. The engine of the car produces a constant horizontal driving force on the car of magnitude $2000\ N$.

a) Find the acceleration of the car and trailer

b) Find the magnitude of the tension in the tow-bar

The engine cuts out, reducing the force produced by the engine to zero and the brakes are applied. The brakes produce a force on the car of magnitude F Newtons and the car and trailer decelerate.

Given that the resistances to motion are unchanged, and the magnitude of the thrust in the towbar is $300\ N$, find the value of F

Your turn

A car of mass $600\ kg$ pulls a trailer of mass $200\ kg$ along a straight horizontal road using a light tow-bar which is parallel to the road. The horizontal resistances to motion of the car and the trailer have magnitudes $300\ N$ and $100\ N$ respectively. The engine of the car produces a constant horizontal driving force on the car of magnitude $1600\ N$.

a) Find the acceleration of the car and trailer

b) Find the magnitude of the tension in the tow-bar

The engine cuts out, reducing the force produced by the engine to zero and the brakes are applied. The brakes produce a force on the car of magnitude F Newtons and the car and trailer decelerate.

Given that the resistances to motion are unchanged, and the magnitude of the thrust in the towbar is $200\ N$, find the value of F

a) $a = 1.5\ ms^{-2}$

b) $400\ N$

c) $800\ N$

10.6) Pulleys

Chapter CONTENTS

Worked example

Particles P and Q, of masses $5m$ and $4m$, are attached to the ends of a light inextensible string. The string passes over a small smooth fixed pulley and the masses hang with the string taut. The system is released from rest.

(a) Write down an equation of motion for P and for Q.
(b) Find the acceleration of each mass.
(c) Find the tension in the string.
(d) Find the force exerted on the pulley by the string.
(e) Find the distance moved by P in the first 2 s, assuming that Q does not reach the pulley.

Your turn

Particles P and Q, of masses $2m$ and $3m$, are attached to the ends of a light inextensible string. The string passes over a small smooth fixed pulley and the masses hang with the string taut. The system is released from rest.

(a) Write down an equation of motion for P and for Q.
(b) Find the acceleration of each mass.
(c) Find the tension in the string.
(d) Find the force exerted on the pulley by the string.
(e) Find the distance moved by Q in the first 4 s, assuming that P does not reach the pulley.

a) For P, $R(\uparrow): T - 2mg = 2ma$
For Q, $R(\downarrow): 3mg - T = 3ma$

b) $a = \frac{1}{5}g = 2.0\ ms^{-2}$ (2 sf)

c) $T = \frac{12}{5}mg\ N$

d) $\frac{24}{5}mg\ N$

e) $15.7\ m$ (3 sf)

Worked example

Two particles A and B of masses 0.8kg and 1.6kg respectively are connected by a light inextensible string. Particle A lies on a rough horizontal table 9m from a small smooth pulley which is fixed at the edge of the table. The string passes over the pulley and B hangs freely, with the string taut, 1m above horizontal ground. A frictional force of magnitude 0.16g opposes the motion of particle A. The system is released from rest. Find:

(a) The acceleration of the system

(b) The time taken for B to reach the ground

(c) The total distance travelled by A before it first comes to rest.

Your turn

Two particles A and B of masses 0.4kg and 0.8kg respectively are connected by a light inextensible string. Particle A lies on a rough horizontal table 4.5m from a small smooth pulley which is fixed at the edge of the table. The string passes over the pulley and B hangs freely, with the string taut, 0.5m above horizontal ground. A frictional force of magnitude 0.08g opposes the motion of particle A. The system is released from rest. Find:

(a) The acceleration of the system

(b) The time taken for B to reach the ground

(c) The total distance travelled by A before it first comes to rest.

a) $0.6g = 5.9\ ms^{-2}$ (2 sf)

b) $0.41\ s$ (2 sf)

c) $2.0\ m$ (2 sf)

Worked example

Two particles A and B have masses $10m$ and km respectively, where $k < 10$. The particles are connected by a light inextensible string which passes over a smooth light fixed pulley. The system is held at rest with the string taut, the hanging parts of the string vertical and with A and B at the same height above a horizontal plane.
The system is released from rest.

After release, A descends with acceleration $\frac{1}{2}g$.

After descending for 2.4 s, the particle A reaches the plane.
It is immediately brought to rest by the impact with the plane.
The initial distance between B and the pulley is such that, in the subsequent motion, B does not reach the pulley.
Find the greatest height reached by B above the plane.

Your turn

Two particles A and B have masses $5m$ and km respectively, where $k < 5$. The particles are connected by a light inextensible string which passes over a smooth light fixed pulley. The system is held at rest with the string taut, the hanging parts of the string vertical and with A and B at the same height above a horizontal plane.
The system is released from rest.

After release, A descends with acceleration $\frac{1}{4}g$.

After descending for 1.2 s, the particle A reaches the plane.
It is immediately brought to rest by the impact with the plane.
The initial distance between B and the pulley is such that, in the subsequent motion, B does not reach the pulley.
Find the greatest height reached by B above the plane.

$4.0\ m$