

# 1) Algebraic methods

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## 1.2) Algebraic fractions

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## Worked example

Simplify:

$$\frac{x^2 - x}{y^2} \div \frac{x^2 + x - 2}{y^5}$$

## Your turn

Simplify:

$$\frac{x^2 + x}{y} \div \frac{x^2 - x - 2}{y^2}$$
$$\frac{xy}{x - 2}$$

## Worked example

Simplify:

$$\frac{10x + 4}{3x^2 + 4x + 1} - \frac{3}{x + 1}$$

## Your turn

Simplify:

$$\frac{3x + 5}{x^2 + x - 12} - \frac{2}{x - 3}$$
$$\frac{1}{x + 4}$$

## Worked example

Simplify:

$$\frac{x^2 + 6x + 5}{x^2 + 3x - 10}$$

$$\frac{x^2 - 7x - 8}{x^2 - 1}$$

## Your turn

Simplify:

$$\frac{x^2 - 6x + 5}{x^2 + 4x - 5}$$
$$\frac{x - 5}{x + 5}$$

## Worked example

Simplify:

$$\frac{2x^2 - 5x - 3}{3x^2 - 11x + 6}$$

$$\frac{3x^2 - x - 10}{x^2 - 4}$$

## Your turn

Simplify:

$$\frac{3x^2 - 5x - 2}{2x^2 - 7x + 6}$$
$$\frac{3x + 1}{2x - 3}$$

## Worked example

Simplify:

$$\frac{3x^2 + 2x - 8}{6x^2 - 23x - 35} \times \frac{7x^2 - 29x - 30}{4x^2 + 5x - 6}$$

$$\frac{6x^2 + 37x - 35}{3x^2 - 10x - 8} \times \frac{3x^2 + 14x + 8}{42x^2 + 35x}$$

## Your turn

Simplify:

$$\frac{2x^2 + 3x - 35}{3x^2 - 11x - 4} \times \frac{6x^2 - 23x - 4}{2x^2 - 3x - 27}$$

$$\frac{(x + 5)(2x - 7)(6x + 1)}{(3x + 1)(2x - 9)(x + 3)}$$

## Worked example

Simplify:

$$\frac{3x^2 - 10x - 8}{6x^2 + 37x - 35} \div \frac{x^2 - 3x - 4}{x^2 - 49}$$

## Your turn

Simplify:

$$\frac{2x^2 - 7x - 15}{3x^2 + 10x - 8} \div \frac{2x^2 + x - 3}{x^2 - 16}$$

$$\frac{(x - 5)(x - 4)}{(3x - 2)(x - 1)}$$



## Worked example

Simplify:

$$\frac{2x^3 - 5x^2 - 3x}{2x - 6}$$

## Your turn

Simplify:

$$\frac{3x^3 - x^2 - 10x}{4x - 8}$$
$$\frac{x(3x + 5)}{4}$$

## Worked example

Simplify:

$$\frac{2x^3 + 5x^2 - 3x}{4x^2 - 1}$$

## Your turn

Simplify:

$$\frac{3x^3 - x^2 - 10x}{9x^2 - 25}$$
$$\frac{x(x - 2)}{3x - 5}$$

## Worked example

Write as a single fraction:

$$\frac{5x + 2}{3} + \frac{x - 3}{2}$$

$$\frac{x - 5}{4} + \frac{2x + 1}{3}$$

## Your turn

Simplify:

$$\frac{4x + 5}{2} + \frac{x - 1}{3}$$
$$\frac{14x + 13}{6}$$

## Worked example

Write as a single fraction:

$$\frac{5x + 2}{3} + \frac{2}{x - 3}$$

$$\frac{x - 5}{4} + \frac{3}{2x + 1}$$

## Your turn

Simplify:

$$\frac{4x + 5}{2} + \frac{3}{x - 1}$$
$$\frac{4x^2 + x + 1}{2(x - 1)}$$

## Worked example

Write as a single fraction:

$$\frac{3}{5x + 2} + \frac{2}{x - 3}$$

$$\frac{4}{x - 5} + \frac{3}{2x + 1}$$

## Your turn

Simplify:

$$\frac{2}{4x + 5} + \frac{3}{x - 1}$$
$$\frac{10x + 13}{(4x + 5)(x - 1)}$$

## Worked example

Write as a single fraction:

$$\frac{5x + 2}{3} - \frac{x - 3}{2}$$

$$\frac{x - 5}{4} - \frac{2x + 1}{3}$$

## Your turn

Simplify:

$$\frac{4x + 5}{2} - \frac{x - 1}{3}$$
$$\frac{10x + 17}{6}$$

## Worked example

Write as a single fraction:

$$\frac{5x + 2}{3} - \frac{2}{x - 3}$$

$$\frac{x - 5}{4} - \frac{3}{2x + 1}$$

## Your turn

Simplify:

$$\frac{4x + 5}{2} - \frac{3}{x - 1}$$
$$\frac{4x^2 + x - 11}{2(x - 1)}$$

## Worked example

Write as a single fraction:

$$\frac{3}{5x + 2} - \frac{2}{x - 3}$$

$$\frac{4}{x - 5} - \frac{3}{2x + 1}$$

## Your turn

Simplify:

$$\frac{2}{4x + 5} - \frac{3}{x - 1}$$
$$\frac{-10x - 17}{(4x + 5)(x - 1)}$$



## Worked example

Write as a single fraction:

$$\frac{3}{(5x + 2)(x - 3)} - \frac{2}{x - 3}$$

## Your turn

Simplify:

$$\frac{2}{4x - 5} - \frac{3}{(x - 1)(4x - 5)}$$

$$\frac{2x - 5}{(x - 1)(4x - 5)}$$

$$\frac{4}{(2x + 1)(x - 5)} - \frac{3}{2x + 1}$$

## Worked example

Write as a single fraction:

$$3 + \frac{5}{2x - 1}$$

$$2 - \frac{3}{5x + 4}$$

## Your turn

Write as a single fraction:

$$5 - \frac{3}{x + 2}$$

$$\frac{5x + 7}{x + 2}$$

## Worked example

Write as a single fraction:

$$3 - (x - 4) \div \frac{x^2 - 16}{x - 5}$$

## Your turn

Write as a single simplified fraction:

$$5 - (x - 2) \div \frac{x^2 - 4}{x + 3}$$

$$\frac{4x - 13}{x - 2}$$

## Worked example

Write in the form  $1 + \frac{a}{x+b}$ :

$$\frac{x+3}{x-5}$$

$$\frac{x-2}{x+7}$$

## Your turn

Write in the form  $1 + \frac{a}{x+b}$ :

$$\frac{x-5}{x+2}$$

$$1 - \frac{7}{x+2}$$

## 1.3) Partial fractions

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## Worked example

Split into partial fractions:

$$\frac{2x - 6}{(x + 3)(x - 1)}$$

## Your turn

Split into partial fractions:

$$\frac{6x - 2}{(x - 3)(x + 1)}$$

$$\frac{4}{x - 3} + \frac{2}{x + 1}$$

## Worked example

Given that  $\frac{-6x^2-5x+2}{x(x-1)(2x+1)} \equiv \frac{A}{x} + \frac{B}{x-1} + \frac{C}{2x+1}$ ,  
find the values of the constants  $A, B, C$ .

## Your turn

Given that  $\frac{6x^2+5x-2}{x(x-1)(2x+1)} \equiv \frac{A}{x} + \frac{B}{x-1} + \frac{C}{2x+1}$ ,  
find the values of the constants  $A, B, C$ .

$$A = 2, B = 3, C = -4$$

## Worked example

Express as partial fractions:

$$\frac{6x^2 + 14x - 12}{x^3 - 4x}$$

## Your turn

Express as partial fractions:

$$\frac{6x^2 + 7x - 3}{x^3 - x}$$

$$\frac{3}{x} - \frac{2}{x+1} + \frac{5}{x-1}$$



## 1.4) Repeated factors

## Worked example

Express as partial fractions:

$$\frac{11x^2 - 22x + 9}{(x - 1)^2(2x - 1)}$$

## Your turn

Express as partial fractions:

$$\frac{11x^2 + 14x + 5}{(x + 1)^2(2x + 1)}$$

$$\frac{4}{x + 1} - \frac{2}{(x + 1)^2} + \frac{3}{2x + 1}$$

## Worked example

Express as partial fractions:

$$\frac{5x^2 + 4x + 1}{x^3 + x^2}$$

## Your turn

Express as partial fractions:

$$\frac{5x^2 - x - 1}{x^3 - x^2}$$

$$\frac{2}{x} + \frac{1}{x^2} + \frac{3}{x-1}$$

## Worked example

Express as partial fractions:

$$\frac{4x}{(x-4)^2}$$

## Your turn

Express as partial fractions:

$$\frac{3x}{(x+3)^2}$$

$$\frac{3}{x+3} - \frac{9}{(x+3)^2}$$

## Worked example

Express as partial fractions:

$$\frac{4x + 1}{x^2 - 8x + 16}$$

## Your turn

Express as partial fractions:

$$\frac{2x + 3}{x^2 + 6x + 9}$$

$$\frac{2}{x + 3} - \frac{3}{(x + 3)^2}$$

## Worked example

Express as partial fractions:

$$\frac{15x^2 - 5x + 2}{9x^3 - 6x^2 + x}$$

## Your turn

Express as partial fractions:

$$\frac{22x^2 + 25x + 1}{4x^3 + 4x^2 + x}$$

$$\frac{6}{x} - \frac{1}{2x + 1} + \frac{2}{(2x + 1)^2}$$

## 1.5) Algebraic division

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## Worked example

Find the values of  $A$ ,  $B$  and  $C$ :

$$\frac{x^2 + 9x - 5}{x + 3} = Ax + B + \frac{C}{x + 3}$$

## Your turn

Find the values of  $A$ ,  $B$  and  $C$ :

$$\frac{x^2 + 5x - 9}{x + 2} = Ax + B + \frac{C}{x + 2}$$

$$A = 1, B = 3, C = -15$$



## Worked example

Find the values of  $A$ ,  $B$  and  $C$ :

$$\frac{x^2 - 5x + 9}{x - 2} = Ax + B + \frac{C}{x - 2}$$

## Your turn

Find the values of  $A$ ,  $B$  and  $C$ :

$$\frac{x^2 + 5x - 9}{x - 3} = Ax + B + \frac{C}{x - 3}$$

$$A = 1, B = 8, C = 15$$

## Worked example

Find the values of  $A$ ,  $B$ ,  $C$  and  $D$ :

$$\frac{x^3 - x^2 + 7}{x + 3} = Ax^2 + Bx + C + \frac{D}{x + 3}$$

## Your turn

Find the values of  $A$ ,  $B$ ,  $C$  and  $D$ :

$$\frac{x^3 + x^2 - 7}{x + 2} = Ax^2 + Bx + C + \frac{D}{x + 2}$$

$$A = 1, B = -1, C = 2, D = -11$$

## Worked example

Find the values of  $A$ ,  $B$ ,  $C$  and  $D$ :

$$\frac{x^3 - x^2 + 7}{x - 2} = Ax^2 + Bx + C + \frac{D}{x - 2}$$

## Your turn

Find the values of  $A$ ,  $B$ ,  $C$  and  $D$ :

$$\frac{x^3 + x^2 - 7}{x - 3} = Ax^2 + Bx + C + \frac{D}{x - 3}$$

$$A = 1, B = 4, C = 12, D = 29$$

## Worked example

Find the values of  $A, B, C, D$  and  $E$ :

$$\frac{x^4 - x^3 - x + 10}{x^2 - 2x - 3} = Ax^2 + Bx + C + \frac{Dx + E}{x^2 + 2x - 3}$$

## Your turn

Find the values of  $A, B, C, D$  and  $E$ :

$$\frac{x^4 + x^3 + x - 10}{x^2 + 2x - 3} = Ax^2 + Bx + C + \frac{Dx + E}{x^2 + 2x - 3}$$

$$A = 1, B = -1, C = 5, D = -12, E = 5$$

## Worked example

Find the values of  $A, B, C$  and  $D$

$$\frac{2x^3 + 3x^2 - 4x + 5}{x - 3} = Ax^2 + Bx + C + \frac{D}{x - 3}$$

## Your turn

Find the values of  $A, B, C$  and  $D$

$$\frac{2x^3 - 3x^2 + 4x - 5}{x + 3} = Ax^2 + Bx + C + \frac{D}{x + 3}$$

$$A = 2, B = -9, C = 31, D = -160$$

## Worked example

Find the values of  $A, B, C, D$  and  $E$ :

$$\frac{3x^4 - 2x^3 + 5x^2 - 4}{x^2 - 16} = Ax^2 + Bx + C + \frac{Dx + E}{x^2 - 16}$$

## Your turn

Find the values of  $A, B, C, D$  and  $E$ :

$$\frac{3x^4 - 2x^3 - 5x^2 - 4}{x^2 - 4} = Ax^2 + Bx + C + \frac{Dx + E}{x^2 - 4}$$

$$A = 3, B = -2, C = 7, D = -8, E = 24$$

## Worked example

Simplify:

$$\frac{x^4 - 81}{x + 3}$$

## Your turn

Simplify:

$$\frac{x^4 - 16}{x + 2}$$

$$(x - 1)(x^2 + 1)$$

## Worked example

Split into partial fractions:

$$\frac{3x^2 - 3x - 2}{(x + 1)(x - 2)}$$

## Your turn

Split into partial fractions:

$$\frac{3x^2 - 3x - 2}{(x - 1)(x - 2)}$$

$$3 + \frac{2}{x - 1} + \frac{4}{x - 2}$$



## Worked example

Split into partial fractions:

$$\frac{9x^2 + 20x - 10}{(x - 2)(3x + 1)}$$

## Your turn

Split into partial fractions:

$$\frac{9x^2 + 20x - 10}{(x + 2)(3x - 1)}$$

$$3 + \frac{2}{x + 2} - \frac{1}{3x - 1}$$

## Worked example

Split into partial fractions:

$$\frac{9x^2 + 16}{9x^2 - 16}$$

## Your turn

Split into partial fractions:

$$\frac{16x^2 + 9}{16x^2 - 9}$$

$$1 - \frac{3}{4x + 3} + \frac{3}{4x - 3}$$

## Worked example

Split into partial fractions:

$$\frac{16x^3 - 36x^2 + 4x + 22}{4x^2 - 12x + 9}$$

## Your turn

Split into partial fractions:

$$\frac{18x^3 - 15x^2 + 5x + 2}{9x^2 - 12x + 4}$$

$$2x + 1 + \frac{3}{3x - 2} + \frac{4}{(3x - 2)^2}$$

## Worked example

Split into partial fractions:

$$\frac{5x^4 - 9x^2 - 7x^2 + 4x - 10}{x^2 - x - 2}$$

## Your turn

Split into partial fractions:

$$\frac{5x^4 + 9x^3 - 6x^2 - 2x - 6}{x^2 + x - 2}$$

$$5x^2 + 4x + 3 + \frac{2}{x + 2} + \frac{1}{x - 1}$$