1) Algebraic expressions

1.1) Index laws
1.4) Negative and fractional indices

1.1) Index Laws

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Worked example	Your turn
Simplify: $(a^5)^3 \times 4a^7$	Simplify: $(a^3)^2 \times 2a^2$ $2a^8$
$(5xy^4)^3$	$(4x^3y)^3$ $64x^9y^3$

Worked example	Your turn
Simplify: $7x^3(6-5x)-4x(3-2x^3)$	Simplify: $2x^2(3+5x)-x(4-x^2)$
	$11x^3+6x^2-4x$
$\dfrac{5x^4+3x^2}{6x^3}$	$\dfrac{x^3-2x}{3x^2}$
	$\dfrac{1}{3}x-\dfrac{2}{3x}$

Worked example	Your turn
Simplify: $4^x \times 5^x$	Simplify: $2^{x-4} \times 3^{x-4}$
	6^{x-4}
$7^{x+1} \times 6^{x+1}$	

Worked example	Your turn
Simplify:	Simplify:
$(x^2)^3$	$(5a^7bc^5)^3$
	$125a^{21}b^3c^{15}$
$(2x^3y^4)^5$	
$(4ab^2c^4)^3$	

Worked example	Your turn
Simplify: $\sqrt{16x^2y^6z^4}$	Simplify: $\sqrt{9a^3b^6c^2d}$ $3a^{\frac{3}{2}}b^3cd^{\frac{1}{2}}$
$\sqrt[3]{27x^4y^6z}$	

Worked example	Your turn
Write in index form: 32×128 $3 \times 27 \times 81$	Write in index form: $5 \times 25 \times 625$ 5^7

Worked example	Your turn
Write 25^3 as 5^n	Write 27^4 as 3^n
	3^{12}
Write 16^5 as 2^n	

Worked example	Your turn
Write $81^4 \times 9^7$ as 3^n	Write $25^{\frac{1}{3}} \times 125^{-\frac{2}{5}}$ as 5^n
	$5^{-\frac{8}{15}}$
Write $8^{\frac{1}{2}} \times 128^{-\frac{3}{4}}$ as 2^n	

Worked example	Your turn
Write $0.04^5 \times 0.2^3$ as 5^n	Write $0.125^3 \times 0.5^7$ as 2^n 2^{-16}

Worked example	Your turn
Write $\frac{81^4}{9^7}$ as 3^n	Write $\frac{25^4}{125^6}$ as 5^n 5^{-10}
Write $\frac{16^3}{32^7}$ as 2^n	

1.4) Negative and fractional indices

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Worked example

Prove that:

$$x^{\frac{1}{2}} = \sqrt{x}$$

$$x^{\frac{1}{3}} = \sqrt[3]{x}$$

Your turn

Prove that:

$$x^{\frac{1}{4}} = \sqrt[4]{x}$$

Proof

Worked example

Evaluate:

$$64^{-\frac{2}{3}}$$

$$81^{-\frac{5}{4}}$$

Your turn

Evaluate:

$$64^{-\frac{3}{2}}$$

$$\frac{1}{512}$$

Worked example

Given that $y = \frac{1}{125}x^3$ express the following in the form kx^n where k and n are constants

a) $y^{\frac{1}{3}}$

b) $5y^{-2}$

Your turn

Given that $y = \frac{1}{16}x^2$ express the following in the form kx^n where k and n are constants:

a) $y^{\frac{1}{2}}$

b) $4y^{-1}$

a) $\frac{x}{4}$

b) $64x^{-2}$

Worked example	Your turn
If $125\sqrt{5} = 5^k$, determine the value of k.	If $9\sqrt{3} = 3^k$, determine the value of k.
	$k = \frac{5}{2}$

Worked example

Given that

$$\left(3^{\frac{1}{4}}\right)^{n} = \frac{3^{x}}{81^{y}}$$

Express n in terms of x and y.

Your turn

Given that

$$\left(2^{\frac{1}{2}}\right)^{n} = \frac{2^{x}}{8^{y}}$$

Express n in terms of x and y.

$$n = 2x - 6y$$

Worked example

Given that $y = 3^x$, express the following in terms of y.

$$\frac{1}{27^{5x-2}}$$

Write your expression in its simplest form.

Your turn

Given that $y = 2^x$, express the following in terms of y.

$$\frac{1}{4^{2x-3}}$$

Write your expression in its simplest form.

$$\frac{64}{y^4}$$

Worked example	Your turn
Simplify:	Simplify:
$(16x^2y^6z^4)^{\frac{1}{2}}$	$(9a^3b^6c^2d)^{\frac{1}{2}}$
	$3a^{\frac{3}{2}}b^3cd^{\frac{1}{2}}$
$(27x^4y^6z)^{\frac{1}{3}}$	

Worked example	Your turn
Convert to fraction form:	Convert to fraction form:
9^{-1}	$5b^{-2}$
5^{-2}	$\frac{5}{b^2}$
2^{-3}	
x^{-5}	
$3y^{-4}$	

Worked example	Your turn
Convert to index form:	Convert to index form:
$\frac{1}{2}$	$\frac{7}{x^3}$
	$7x^{-3}$
$\frac{1}{100}$	
$\frac{3}{x}$	$\frac{12}{\sqrt[3]{x}}$
	$12x^{-\frac{1}{3}}$
$\frac{9}{x^2}$	
$\frac{15}{\sqrt{x}}$	

Worked example

Evaluate:

2^4

3^5

4^0

5^{-1}

6^{-2}

7^{-3}

$9^{\frac{1}{2}}$

$8^{\frac{1}{3}}$

$16^{-\frac{1}{2}}$

$125^{-\frac{1}{3}}$

$25^{\frac{3}{2}}$

$64^{-\frac{2}{3}}$

Your turn

Evaluate:

$36^{-\frac{1}{2}}$

$\frac{1}{6}$

123456789^0

1

$27^{-\frac{4}{3}}$

$\frac{1}{81}$

Worked example	Your turn
Write $\frac{1}{243}$ as 3^n	Write 0.125 as 2^n 2^{-3}
Write 0.04 as 5^n	

Worked example	Your turn
Evaluate:	Evaluate:
3^{-4}	5^{-3}
	$\frac{1}{125}$
2^{-5}	

Worked example	Your turn
Write as a fraction: x^{-5}	Write as a fraction: z^{-3} $\frac{1}{z^3}$
y^{-4}	

Worked example	Your turn
Write as a fraction: $3x^{-5}$ $5y^{-4}$	Write as a fraction: $2z^{-3}$ $\frac{2}{z^3}$

Worked example	Your turn
Simplify the following: $(r^{-3})^4$ $(s^5)^{-2}$	Simplify the following: $(t^{-4})^5$ t^{-20}

Worked example	Your turn
Simplify the following: $(2r^{-3})^4$ $(3s^{-4})^{-2}$ $(5t^{-2})^3$	Simplify the following: $(3u^{-4})^{-3}$ $\frac{u^{12}}{27}$

Worked example	Your turn
Simplify the following: $(2r^{-3}s^{4})^{5}$ $(3t^{-4}u^{5})^{-2}$	Simplify the following: $(5x^{3}y^{-2})^{-3}$ $\frac{y^{6}}{125x^{9}}$

Worked example	Your turn
Simplify the following: $\frac{15x^7}{3x^{-4}}$ $\frac{56y^{-8}}{8y^{-7}}$	Simplify the following: $\frac{42x^{-5}}{6x^{-3}}$ $7x^{-2} = \frac{7}{x^2}$

Worked example	Your turn
Simplify the following: $\left(\frac{a^2b^{-3}}{c^4}\right)^5$	Simplify the following: $\left(\frac{a^{-4}b^3}{c^2}\right)^3$ $\frac{b^9}{a^{12}c^6}$

Worked example	Your turn
Simplify the following: $(3a^{\frac{4}{5}})^5$	Simplify the following: $(4b^{\frac{2}{3}})^3$ $64b^2$

Worked example	Your turn
Evaluate:	Simplify the following:
$64^{\frac{1}{2}}$	$125^{-\frac{1}{3}}$
	$\frac{1}{5}$
$64^{\frac{1}{3}}$	
$64^{-\frac{1}{2}}$	

Worked example	Your turn
Evaluate:	Simplify the following:
$64^{\frac{3}{2}}$	$125^{-\frac{2}{3}}$
	$\frac{1}{125}$
$64^{\frac{2}{3}}$	
$64^{-\frac{3}{2}}$	

Worked example	Your turn
Evaluate:	Simplify the following:
$81^{\frac{1}{2}}$	$27^{-\frac{1}{3}}$
	$\frac{1}{3}$
$81^{\frac{1}{4}}$	
$81^{-\frac{1}{2}}$	

Worked example	Your turn
Evaluate:	Simplify the following:
$81^{\frac{3}{2}}$	$27^{-\frac{4}{3}}$
	$\frac{1}{81}$
$81^{\frac{3}{4}}$	
$81^{-\frac{3}{2}}$	

Worked example	Your turn
Evaluate: $\left(\frac{32}{243}\right)^{-\frac{3}{5}}$	Simplify the following: $\left(\frac{64}{125}\right)^{-\frac{2}{3}}$ $\frac{25}{16}$

Worked example	Your turn
Write in index form: $\sqrt[3]{25}$ $\sqrt{27}$	Write in index form: $\sqrt[4]{32}$ $2^{\frac{5}{4}}$

Worked example	Your turn
Write in index form: $\frac{1}{\sqrt[3]{25}}$	Write in index form: $\frac{1}{\sqrt[4]{32}}$ $2^{-\frac{5}{4}}$
$\frac{1}{\sqrt{27}}$	

Worked example	Your turn
Simplify: $(64a^6)^{\frac{3}{2}}$	Simplify: $(27b^6)^{\frac{2}{3}}$
$(64a^6)^{\frac{2}{3}}$	$9b^4$

Worked example	Your turn
Evaluate: $\left(\sqrt{\frac{3}{2}}\right)^4$	Evaluate: $\left(\sqrt{\frac{3}{8}}\right)^4$
	$\frac{9}{64}$
$\left(\sqrt{\frac{5}{7}}\right)^4$	

Worked example	Your turn
Express 243 as a power of 9	Express 125 as a power of 25 $25^{\frac{3}{2}}$
Express 32 as a power of 4	

Worked example	Your turn
Express in index form: $\sqrt[3]{x^4}$	Express in index form: $\sqrt[4]{x^7}$
$\sqrt{x^5}$	$x^{\frac{7}{4}}$

Worked example	Your turn
Express in index form:	Express in index form:
$\frac{1}{x}$	$\frac{1}{x^4}$
	x^{-4}
$\frac{1}{x^2}$	
$\frac{1}{x^3}$	

Worked example	Your turn
Express in index form:	Express in index form:
$\frac{1}{\sqrt{x}}$	$\frac{1}{\sqrt[7]{x}}$
	$x^{-\frac{1}{7}}$
$\frac{1}{\sqrt[3]{x}}$	
$\frac{1}{\sqrt[5]{x}}$	

Worked example	Your turn
Express in index form:	Express in index form:
$\frac{2}{x}$	$\frac{7}{x^4}$
	$7x^{-4}$
$\frac{3}{x^2}$	
$\frac{5}{x^3}$	

Worked example	Your turn
Express in index form:	Express in index form:
$\frac{2}{\sqrt{x}}$	$\frac{3}{\sqrt[7]{x}}$
	$3x^{-\frac{1}{7}}$
$\frac{5}{\sqrt[3]{x}}$	
$\frac{7}{\sqrt[5]{x}}$	

Worked example	Your turn
Express in index form: $\frac{2}{3\sqrt{x}}$	Express in index form: $\frac{2}{5\sqrt[7]{x}}$ $\frac{2}{5}x^{-\frac{1}{7}}$
$\frac{5}{7\sqrt[3]{x}}$	
$\frac{1}{4\sqrt[5]{x}}$	

Worked example

Simplify fully:

$$\sqrt{a^{\frac{2}{3}} \times a^{\frac{4}{7}}}$$

$$\sqrt{b^{\frac{2}{5}} \times b^{\frac{6}{7}}}$$

Your turn

Simplify fully:

$$\sqrt{a^{\frac{3}{4}} \times a^{\frac{3}{5}}}$$

$$a^{\frac{27}{40}}$$

Worked example	Your turn
Solve:	Solve:
$5^x = 125$	$7^x = 343$
	$x = 3$
$2^x = 16$	
$3^x = 243$	

Worked example	Your turn
Solve: $5^{x-2} = 25$	Solve: $2^{2-5x} = 16$
	$x = -\frac{2}{5}$
$2^{2x+3} = 32$	
$3^{1-3x} = 81$	

Worked example	Your turn
Solve:	Solve:
$5 = 25^x$	$7 = 343^{x-2}$
	$x = \frac{7}{3}$
$3 = 27^x$	
$2 = 16^{x-1}$	

Worked example	Your turn
Solve: $5^x = \frac{1}{125}$	Solve: $7^x = \frac{1}{343}$
	$x = -3$
$3^x = \frac{1}{9}$	
$2^x = \frac{1}{16}$	

Worked example	Your turn
Solve: $5^{-x} = \frac{1}{125}$	Solve: $7^{-x} = \frac{1}{343}$
	$x = 3$
$3^{-x} = \frac{1}{9}$	
$2^{-x} = \frac{1}{16}$	

Worked example	Your turn
Solve: $5^{-x} = 125$	Solve: $7^{-x} = 343$
	$x = -3$
$3^{-x} = 9$	
$2^{-x} = 16$	

Worked example	Your turn
Solve: $125^x = \frac{1}{5}$	Solve: $343^x = \frac{1}{7}$
	$x = -\frac{1}{3}$
$9^x = \frac{1}{3}$	
$8^x = \frac{1}{2}$	

Worked example	Your turn
Solve: $2^{3x-2} = \frac{1}{64}$	Solve: $5^{3-2x} = \frac{1}{25}$
	$x = \frac{5}{2}$
$3^{2-5x} = \frac{1}{81}$	

Worked example	Your turn
Solve: $2^x = \sqrt[3]{16}$	Solve: $5^{x+1} = \sqrt[4]{25}$
	$x = -\frac{1}{2}$
$3^{x-2} = \sqrt{27}$	

Worked example	Your turn
Solve: $2^{3x-1} = \sqrt[4]{32}$	Solve: $5^{3-2x} = \sqrt[3]{25}$
	$x = \frac{7}{6}$
$3^{2-5x} = \sqrt{243}$	

Worked example	Your turn
Solve: $2^{3x-5} = \frac{16}{\sqrt[5]{64}}$	Solve: $3^{2x-5} = \frac{27}{\sqrt{243}}$ $x = \frac{11}{4}$

Worked example	Your turn
Solve: $16 \times 2^{3x-1} = \frac{1}{\sqrt{32}}$	Solve: $27 \times 3^{5x-2} = \frac{1}{\sqrt[3]{9}}$ $x = -\frac{1}{3}$

Worked example	Your turn
Solve: $(16^x)^3 = \frac{1}{8}$	Solve: $(9^x)^5 = \frac{1}{27}$ $x = -\frac{3}{10}$

Worked example	Your turn
Solve: $\sqrt{2} \times 8^{2x-5} = \frac{1}{16}$	Solve: $\sqrt{3} \times 9^{4x-3} = \frac{1}{27}$ $x = \frac{5}{16}$

Worked example	Your turn
Solve: $\frac{16^{4x-3}}{8^{5-2x}} = 4$	Solve: $\frac{9^{3-5x}}{27^{4x-3}} = 81$ $x = \frac{1}{2}$

Worked example	Your turn
Express y in terms of x, given: $2^x \times 2^y = 8$	Express y in terms of x, given: $5^x \times 5^y = 125$ $y = 3 - x$
$3^x \times 3^y = \sqrt{27}$	

Worked example	Your turn
Express y in terms of x, given: $2^{3x} \times 8^{5y} = \frac{1}{4}$	Express y in terms of x, given: $5^{3x} \times 25^{4y} = \frac{1}{\sqrt{125}}$ $y = -\frac{3}{8}x - \frac{3}{16}$

Worked example	Your turn
Express y in terms of x, given: $\frac{2^{3x}}{2^{5y}} = 2\sqrt{2}$ $\frac{3^{4x}}{3^{2y}} = \frac{1}{27}$	Express y in terms of x, given: $\frac{5^{x}}{5^{4y}} = 125\sqrt{5}$ $y = \frac{x}{4} - \frac{7}{8}$

Worked example	Your turn
Express y in terms of x, given: $\frac{2^{3x}}{8^{5y}} = 2\sqrt{2}$ $\frac{3^{4x}}{9^{2y}} = \frac{1}{27}$	Express y in terms of x, given: $\frac{5^{x}}{25^{4y}} = 125\sqrt{5}$ $y = \frac{x}{8} - \frac{7}{16}$

Worked example	Your turn
Solve $\left(\frac{1}{2}\right)^x = 8$	Solve $\left(\frac{1}{3}\right)^x = 9$ $x = -2$
Solve $25^{3-4x} = \frac{1}{125}$	Solve $9^{4-3x} = \frac{1}{81}$ $x = 2$

Worked example	Your turn
Solve $x^{\frac{1}{2}} = 5$	Solve $x^{\frac{1}{4}} = 2$ $x = 16$
Solve $x^{\frac{1}{3}} = 6$	

Worked example	Your turn
Solve $x^{-2} = 25$	Solve $x^{-4} = 16$
	$x = \frac{1}{2}$
Solve $x^{-3} = 216$	

Worked example	Your turn
Given that $5^{-n} = 0.4$, find the value of $(5^3)^n$	Given that $3^{-n} = 0.2$, find the value of $(3^4)^n$ 625

Worked example	Your turn
Express $\frac{\left(3+\sqrt[3]{x}\right)^2}{x}$ as powers of x.	Express $\frac{\left(2+\sqrt{x}\right)^2}{x}$ as powers of x.
	$4x^{-1}+4x^{-\frac{1}{2}}+1$

Worked example	Your turn
Express $\frac{\left(x+\sqrt[3]{x}\right)^2}{4\sqrt[3]{x}}$ as powers of x.	Express $\frac{\left(x+\sqrt{x}\right)^2}{2\sqrt{x}}$ as powers of x. $\frac{1}{2}x^{\frac{3}{2}}+x+\frac{1}{2}x^{\frac{1}{2}}$