1) Algebraic expressions

1.1) Index Laws

Worked example	Your turn
Simplify:	Simplify:
$(a^5)^3 \times 4a^7$	$(a^3)^2 \times 2a^2$
	$2a^8$
$(5xy^4)^3$	$(4x^3y)^3$
	$64x^9y^3$

Worked example	Your turn
Simplify: $$7x^3(6 - 5x) - 4x(3 - 2x^3)$$	Simplify: $$2x^2(3 + 5x) - x(4 - x^2)$$ $$11x^3 + 6x^2 - 4x$$
$$\frac{5x^4 + 3x^2}{6x^3}$$	$$\frac{x^3 - 2x}{3x^2}$$ $$\frac{1}{3}x - \frac{2}{3x}$$

Worked example	Your turn
Simplify:	Simplify:
$$4^x \times 5^x$$	$$2^{x-4} \times 3^{x-4}$$ $$\textcolor{red}{6^{x-4}}$$
$$7^{x+1} \times 6^{x+1}$$	

Worked example	Your turn
Simplify:	Simplify:
$(x^2)^3$	$(5a^7bc^5)^3$
	$125a^{21}b^3c^{15}$
$(2x^3y^4)^5$	
$(4ab^2c^4)^3$	

Worked example	Your turn
Simplify: $$\sqrt{16x^2y^6z^4}$$	Simplify: $$\sqrt{9a^3b^6c^2d}$$ $$\color{red}{3a^{\frac{3}{2}}b^3cd^{\frac{1}{2}}}$$

$$\sqrt[3]{27x^4y^6z}$$

Worked example	Your turn
Write in index form: 32×128	Write in index form: $5 \times 25 \times 625$ 5^7
$3 \times 27 \times 81$	

Worked example	Your turn
Write 25^3 as 5^n	Write 27^4 as 3^n
	3^{12}
Write 16^5 as 2^n	

Worked example	Your turn
Write $81^4 \times 9^7$ as 3^n	Write $25^{\frac{1}{3}} \times 125^{-\frac{2}{5}}$ as 5^n
	$5^{-\frac{8}{15}}$
Write $8^{\frac{1}{2}} \times 128^{-\frac{3}{4}}$ as 2^n	

Worked example	Your turn
Write $0.04^5 \times 0.2^3$ as 5^n	Write $0.125^3 \times 0.5^7$ as 2^n
	2^{-16}

Worked example	Your turn
Write $\dfrac{81^4}{9^7}$ as 3^n	Write $\dfrac{25^4}{125^6}$ as 5^n
	5^{-10}
Write $\dfrac{16^3}{32^7}$ as 2^n	

1.2) Expanding brackets

Worked example	Your turn
Expand and simplify: $$(3x - 5y)(x - 4y + 2)$$	Expand and simplify: $$(5x - 3y)(2x - y + 4)$$ $$10x^2 - 11xy + 20x + 3y^2 - 12y$$

Worked example	Your turn
Expand and simplify: $x^2(3x - 5y)(x - 4y + 2)$	Expand and simplify: $x(5x - 3y)(2x - y + 4)$ $\color{red}{10x^3 - 11x^2y + 20x^2 + 3xy^2 - 12xy}$

Worked example	Your turn
Expand and simplify: $$(2x - 5)^3$$	Expand and simplify: $$(4x - 7)^3$$ $$64x^3 - 336x^2 + 588x - 343$$

Worked example	Your turn
Expand:	Expand:
$(2x + 3)(4x + 5)$	$(3a + 5)(2a + 4)$
	$6a^2 + 22a + 20$
$(5x + 3)(4x + 1)$	

Worked example	Your turn
Expand:	Expand:
$(x + 3)^2$	$(x + 5)^2$
	$x^2 + 10x + 25$
$(x + 4)^2$	

Worked example	Your turn
Expand:	Expand:
$(x-3)^2$	$(x-5)^2$
	$x^2 - 10x + 25$
$(x-4)^2$	

Worked example	Your turn
Expand:	Expand:
$(4x + 3)(4x - 3)$	$(3x + 5)(3x - 5)$
	$9x^2 - 25$
$(5x + 4)(5x - 4)$	

Worked example	Your turn
Expand:	Expand:
$(3x + 4)^2$	$(4x + 5)^2$
	$16x^2 + 40x + 25$
$(5x + 6)^2$	

Worked example	Your turn
Expand:	Expand:

Worked example:

Expand:

$$(3x - 4)^2$$

$$(5x - 6)^2$$

Your turn:

Expand:

$$(4x - 5)^2$$

$$16x^2 - 40x + 25$$

Worked example	Your turn
Expand:	Expand:
$(4 - 3x)^2$	$(5 - 4x)^2$
	$16x^2 - 40x + 25$
$(6 - 5x)^2$	

Worked example	Your turn
Expand: $$(3x + 2y)^2$$ $$(4x + 3y)^2$$	Expand: $$(5x + 4y)^2$$ $$25x^2 + 40xy + 16y^2$$

Worked example	Your turn
Expand:	Expand:
$2(4x + 3)(5x - 6)$	$4(5x + 6)(3x - 2)$
	$60x^2 + 32x - 48$
$3(7x + 2)(8x - 5)$	

Worked example	Your turn
Expand:	Expand:
$2x(4x + 3)(5x - 6)$	$4x^2(5x + 6)(3x - 2)$
	$60x^4 + 32x^3 - 48x^2$
$3x^2(7x + 2)(8x - 5)$	

Worked example	Your turn
Expand: $(x + 5)(x + 6)(x + 7)$	Expand: $(x + 2)(x + 3)(x + 4)$ $x^3 + 9x^2 + 26x + 24$

Worked example	Your turn
Expand: $(2x + 5)(3x + 6)(4x + 7)$	Expand: $(3x + 2)(4x + 6)(5x + 7)$ $60x^3 + 214x^2 + 242x + 84$

Worked example	Your turn
Expand: $$(x + 5)(x - 6)(x - 7)$$	Expand: $$(x + 2)(x - 3)(x - 4)$$ $$x^3 - 5x^2 - 2x + 24$$

Worked example	Your turn
Expand: $$(x-5)(x-6)(x-7)$$	Expand: $$(x-2)(x-3)(x-4)$$ $$x^3 - 9x^2 + 26x - 24$$

Worked example	Your turn
Expand: $$(x+5)(x+6)^2$$	Expand: $$(x+2)(x+3)^2$$ $$x^3 + 8x^2 + 21x + 18$$

Worked example	Your turn
Expand: $$(x-5)(x-6)^2$$	Expand: $$(x-2)(x-3)^2$$ $$x^3 - 8x^2 + 21x - 18$$

Worked example	Your turn
Expand:	Expand:
$(x + 5)^3$	$(x + 4)^3$
	$x^3 + 12x^2 + 48x + 64$
$(x + 6)^3$	

Worked example	Your turn
Expand:	Expand:
$(x-5)^3$	$(x-4)^3$
	$x^3 - 12x^2 + 48x - 64$
$(x-6)^3$	

Worked example	Your turn
Expand: $$(5 - 2x)(3x - 4)^2$$	Expand: $$(7 - 3x)(4x - 5)^2$$ $$-48x^3 + 232x^2 - 355x + 175$$

1.3) Factorising

Worked example	Your turn
Factorise: $$21x^4 + 29x^3 - 10x^2$$	Factorise: $$10x^3 + 29x^2 - 21x$$ $$x(5x - 3)(2x + 7)$$

Worked example	Your turn
Factorise: $$x^3 - x$$	Factorise: $$x^3 - x$$ $$\textcolor{red}{x(x-1)(x+1)}$$

Worked example	Your turn
Factorise:	Factorise:
$x^2 + 4x$	$3x^2 - 12x$
	$3x(x - 4)$
$x^2 - 6x$	
$2x^2 - 6x$	

Worked example	Your turn
Factorise:	Factorise:

Worked example:

Factorise:

$$x^2 + x - 2$$

$$x^2 - x - 2$$

$$x^2 - x - 6$$

$$x^2 + x - 12$$

Your turn:

Factorise:

$$x^2 - x - 20$$

$$(x - 5)(x + 4)$$

Worked example	Your turn
Factorise: $$2x^2 - 13x + 15$$	Factorise: $$3x^2 - 11x + 10$$ $$\textcolor{red}{(3x - 5)(x - 2)}$$
$$3x^2 - 17x + 10$$	

Worked example	Your turn
Factorise: $$2x^2 + 2x - 12$$	Factorise: $$5x^2 - 15x + 10$$ $$\textcolor{red}{5(x - 2)(x - 1)}$$
$$3x^2 - 12x - 15$$	

Worked example	Your turn
Factorise:	Factorise:
$3x^2 + 10x + 3$	$5x^2 + 8x + 3$
	$(5x + 3)(x + 1)$
$3x^2 + 10x + 8$	$5x^2 + 16x + 12$
	$(5x + 6)(x + 2)$
$3x^2 + 14x + 8$	$5x^2 + 32x + 12$
	$(5x + 2)(x + 6)$

Worked example	Your turn
Factorise: $$(x^2 - y^2) - (x - y)^2$$	Factorise: $$(n^2 - a^2) - (n - a)^2$$ $$2a(n - a)$$

Worked example	Your turn
Factorise: $(x + y)^2 + (x + y)(2x + 5y)$	Factorise: $(p + q)^2 + (p + q)(4p + 9q)$ $5(p + q)(p + 2q)$
$(a + b)^2 - (a + b)(2a - 3b)$	

Worked example	Your turn
Factorise:	Factorise:
$a^2 - 9$	$c^2 - 25$
	$(c + 5)(c - 5)$
$b^2 - 16$	

Worked example	Your turn
Factorise: $$9 - x^2$$ $$16 - x^2$$	Factorise: $$25 - x^2$$ $$(5 + x)(5 - x)$$

Worked example	Your turn
Factorise: $$2x^2 - 18$$	Factorise: $$4x^2 - 100$$ $$\textcolor{red}{4(x + 5)(x - 5)}$$
$$3x^2 - 48$$	

Worked example	Your turn
Factorise:	Factorise:
$$4x^2 - 9$$	$$25x^2 - 36$$
	$$(5x + 6)(5x - 6)$$
$$9x^2 - 16$$	

Worked example	Your turn
Factorise:	Factorise:
$$9x^2 - 1$$	$$25x^2 - 1$$
	$$(5x + 1)(5x - 1)$$
$$1 - 16x^2$$	

Worked example	Your turn
Factorise: $$9x^2 - y^2$$	Factorise: $$16c^2 - d^2$$ $$(4c + d)(4c - d)$$
$$a^2 - 25b^2$$	

Worked example	Your turn
Factorise: $$4x^2 - 9y^2$$ $$9x^2 - 16y^2$$	Factorise: $$25x^2 - 36y^2$$ $$(5x + 6y)(5x - 6y)$$

Worked example	Your turn
Factorise: $$5m^2 - 20p^2$$	Factorise: $$50a^2 - 72b^2$$ $$2(5a + 6b)(5a - 6b)$$
$$32x^2 - 98y^2$$	

Worked example	Your turn
Factorise:	Factorise:
$$x^4 - 1$$	$$x^{10} - 49$$
	$$(x^5 + 7)(x^5 - 7)$$
$$x^6 - 16$$	
$$x^8 - 25$$	

Worked example	Your turn
Factorise: $$4x^4 - 9y^2$$	Factorise: $$25x^{12} - 36y^6$$ $$\color{red}(5x^6 + 6y^3)(5x^6 - 6y^3)$$
$$9x^8 - 16y^6$$	

Worked example	Your turn
Factorise: $$x^3 - 8$$	Factorise: $$x^3 - 1$$ $$\color{red}{(x-1)(x^2 + x + 1)}$$

1.4) Negative and fractional indices

Worked example	Your turn
Prove that: $$x^{\frac{1}{2}} = \sqrt{x}$$	Prove that: $$x^{\frac{1}{4}} = \sqrt[4]{x}$$ Proof
$$x^{\frac{1}{3}} = \sqrt[3]{x}$$	

Worked example	Your turn
Evaluate:	Evaluate:
$$64^{-\frac{2}{3}}$$	$$64^{-\frac{3}{2}}$$
	$$\frac{1}{512}$$
$$81^{-\frac{5}{4}}$$	

Worked example	Your turn
Given that $y = \frac{1}{125}x^3$ express the following in the form kx^n where k and n are constants a) $y^{\frac{1}{3}}$ b) $5y^{-2}$	Given that $y = \frac{1}{16}x^2$ express the following in the form kx^n where k and n are constants: a) $y^{\frac{1}{2}}$ b) $4y^{-1}$ a) $\frac{x}{4}$ b) $64x^{-2}$

Worked example	Your turn
If $125\sqrt{5} = 5^k$, determine the value of k.	If $9\sqrt{3} = 3^k$, determine the value of k.
	$k = \dfrac{5}{2}$

Worked example	Your turn
Given that $$\left(3^{\frac{1}{4}}\right)^n = \frac{3^x}{81^y}$$ Express n in terms of x and y.	Given that $$\left(2^{\frac{1}{2}}\right)^n = \frac{2^x}{8^y}$$ Express n in terms of x and y. $$n = 2x - 6y$$

Worked example	Your turn
Given that $y = 3^x$, express the following in terms of y. $$\frac{1}{27^{5x-2}}$$ Write your expression in its simplest form.	Given that $y = 2^x$, express the following in terms of y. $$\frac{1}{4^{2x-3}}$$ Write your expression in its simplest form. $$\frac{64}{y^4}$$

Worked example	Your turn
Simplify:	Simplify:

Worked example:

$$(16x^2y^6z^4)^{\frac{1}{2}}$$

$$(27x^4y^6z)^{\frac{1}{3}}$$

Your turn:

$$(9a^3b^6c^2d)^{\frac{1}{2}}$$

$$3a^{\frac{3}{2}}b^3cd^{\frac{1}{2}}$$

Worked example	Your turn
Convert to fraction form:	Convert to fraction form:
9^{-1}	$5b^{-2}$
5^{-2}	$\dfrac{5}{b^2}$
2^{-3}	
x^{-5}	
$3y^{-4}$	

Worked example	Your turn
Convert to index form:	Convert to index form:

Worked example:

$$\frac{1}{2}$$

$$\frac{1}{100}$$

$$\frac{3}{x}$$

$$\frac{9}{x^2}$$

$$\frac{15}{\sqrt{x}}$$

Your turn:

$$\frac{7}{x^3}$$

$$7x^{-3}$$

$$\frac{12}{\sqrt[3]{x}}$$

$$12x^{-\frac{1}{3}}$$

Worked example	Your turn
Evaluate:	Evaluate:

Worked example

Evaluate:

2^4 $\qquad\qquad$ 3^5

4^0 $\qquad\qquad$ 5^{-1}

6^{-2} $\qquad\qquad$ 7^{-3}

$9^{\frac{1}{2}}$ $\qquad\qquad$ $8^{\frac{1}{3}}$

$16^{-\frac{1}{2}}$ $\qquad\qquad$ $125^{-\frac{1}{3}}$

$25^{\frac{3}{2}}$ $\qquad\qquad$ $64^{-\frac{2}{3}}$

Your turn

Evaluate:

$36^{-\frac{1}{2}}$

$\dfrac{1}{6}$

123456789^0

1

$27^{-\frac{4}{3}}$

$\dfrac{1}{81}$

Worked example	Your turn
Write $\frac{1}{243}$ as 3^n	Write 0.125 as 2^n
	2^{-3}
Write 0.04 as 5^n	

Worked example	Your turn
Evaluate:	Evaluate:
3^{-4}	5^{-3}
	$\dfrac{1}{125}$
2^{-5}	

Worked example	Your turn
Write as a fraction: $$x^{-5}$$ $$y^{-4}$$	Write as a fraction: $$z^{-3}$$ $$\frac{1}{z^3}$$

Worked example	Your turn
Write as a fraction: $$3x^{-5}$$ $$5y^{-4}$$	Write as a fraction: $$2z^{-3}$$ $$\frac{2}{z^3}$$

Worked example	Your turn
Simplify the following: $(r^{-3})^4$	Simplify the following: $(t^{-4})^5$ t^{-20}
$(s^5)^{-2}$	

Worked example	Your turn
Simplify the following:	Simplify the following:
$(2r^{-3})^4$	$(3u^{-4})^{-3}$
	$\dfrac{u^{12}}{27}$
$(3s^{-4})^{-2}$	
$(5t^{-2})^3$	

Worked example	Your turn
Simplify the following: $$(2r^{-3}s^4)^5$$	Simplify the following: $$(5x^3y^{-2})^{-3}$$ $$\frac{y^6}{125x^9}$$
$$(3t^{-4}u^5)^{-2}$$	

Worked example	Your turn
Simplify the following: $$\frac{15x^7}{3x^{-4}}$$	Simplify the following: $$\frac{42x^{-5}}{6x^{-3}}$$ $$7x^{-2} = \frac{7}{x^2}$$
$$\frac{56y^{-8}}{8y^{-7}}$$	

Worked example	Your turn
Simplify the following: $$\left(\frac{a^2 b^{-3}}{c^4}\right)^5$$	Simplify the following: $$\left(\frac{a^{-4} b^3}{c^2}\right)^3$$ $$\frac{b^9}{a^{12} c^6}$$

Worked example	Your turn
Simplify the following: $$\left(3a^{\frac{4}{5}}\right)^5$$	Simplify the following: $$\left(4b^{\frac{2}{3}}\right)^3$$ $$64b^2$$

Worked example	Your turn
Evaluate:	Simplify the following:
$64^{\frac{1}{2}}$	$125^{-\frac{1}{3}}$
	$\dfrac{1}{5}$
$64^{\frac{1}{3}}$	
$64^{-\frac{1}{2}}$	

Worked example	Your turn
Evaluate:	Simplify the following:

Worked example:

$$64^{\frac{3}{2}}$$

$$64^{\frac{2}{3}}$$

$$64^{-\frac{3}{2}}$$

Your turn:

$$125^{-\frac{2}{3}}$$

$$\frac{1}{125}$$

Worked example	Your turn
Evaluate:	Simplify the following:
$$81^{\frac{1}{2}}$$	$$27^{-\frac{1}{3}}$$ $$\frac{1}{3}$$
$$81^{\frac{1}{4}}$$	
$$81^{-\frac{1}{2}}$$	

Worked example	Your turn
Evaluate:	Simplify the following:
$$81^{\frac{3}{2}}$$	$$27^{-\frac{4}{3}}$$
	$$\frac{1}{81}$$
$$81^{\frac{3}{4}}$$	
$$81^{-\frac{3}{2}}$$	

Worked example	Your turn
Evaluate:	Simplify the following:
$$\left(\frac{32}{243}\right)^{-\frac{3}{5}}$$	$$\left(\frac{64}{125}\right)^{-\frac{2}{3}}$$
	$$\frac{25}{16}$$

Worked example	Your turn
Write in index form: $\sqrt[3]{25}$ $\sqrt{27}$	Write in index form: $\sqrt[4]{32}$ $2^{\frac{5}{4}}$

Worked example	Your turn
Write in index form:	Write in index form:
$$\dfrac{1}{\sqrt[3]{25}}$$	$$\dfrac{1}{\sqrt[4]{32}}$$
	$$2^{-\frac{5}{4}}$$
$$\dfrac{1}{\sqrt{27}}$$	

Worked example	Your turn
Simplify:	Simplify:
$(64a^6)^{\frac{3}{2}}$	$(27b^6)^{\frac{2}{3}}$
	$9b^4$
$(64a^6)^{\frac{2}{3}}$	

Worked example	Your turn
Evaluate:	Evaluate:
$$\left(\sqrt{\frac{3}{2}}\right)^4$$	$$\left(\sqrt{\frac{3}{8}}\right)^4$$ $$\frac{9}{64}$$
$$\left(\sqrt{\frac{5}{7}}\right)^4$$	

Worked example	Your turn
Express 243 as a power of 9	Express 125 as a power of 25
	$25^{\frac{3}{2}}$
Express 32 as a power of 4	

Worked example	Your turn
Express in index form: $$\sqrt[3]{x^4}$$	Express in index form: $$\sqrt[4]{x^7}$$ $$x^{\frac{7}{4}}$$
$$\sqrt{x^5}$$	

Worked example	Your turn
Express in index form:	Express in index form:
$$\frac{1}{x}$$	$$\frac{1}{x^4}$$
	x^{-4}
$$\frac{1}{x^2}$$	
$$\frac{1}{x^3}$$	

Worked example	Your turn
Express in index form:	Express in index form:
$$\dfrac{1}{\sqrt{x}}$$	$$\dfrac{1}{\sqrt[7]{x}}$$ $$x^{-\frac{1}{7}}$$
$$\dfrac{1}{\sqrt[3]{x}}$$	
$$\dfrac{1}{\sqrt[5]{x}}$$	

Worked example	Your turn
Express in index form:	Express in index form:
$$\frac{2}{x}$$	$$\frac{7}{x^4}$$
	$7x^{-4}$
$$\frac{3}{x^2}$$	
$$\frac{5}{x^3}$$	

Worked example	Your turn
Express in index form:	Express in index form:
$$\dfrac{2}{\sqrt{x}}$$	$$\dfrac{3}{\sqrt[7]{x}}$$ $$\textcolor{red}{3x^{-\frac{1}{7}}}$$
$$\dfrac{5}{\sqrt[3]{x}}$$	
$$\dfrac{7}{\sqrt[5]{x}}$$	

Worked example	Your turn
Express in index form:	Express in index form:
$$\frac{2}{3\sqrt{x}}$$	$$\frac{2}{5\sqrt[7]{x}}$$
	$$\frac{2}{5}x^{-\frac{1}{7}}$$
$$\frac{5}{7\sqrt[3]{x}}$$	
$$\frac{1}{4\sqrt[5]{x}}$$	

Worked example	Your turn
Simplify fully:	Simplify fully:

$$\sqrt{a^{\frac{2}{3}} \times a^{\frac{4}{7}}}$$

$$\sqrt{a^{\frac{3}{4}} \times a^{\frac{3}{5}}}$$

$a^{\frac{27}{40}}$

$$\sqrt{b^{\frac{2}{5}} \times b^{\frac{6}{7}}}$$

Worked example	Your turn
Solve:	Solve:
$$5^x = 125$$	$$7^x = 343$$
	$$x = 3$$
$$2^x = 16$$	
$$3^x = 243$$	

Worked example	Your turn
Solve:	Solve:
$$5^{x-2} = 25$$	$$2^{2-5x} = 16$$
	$$x = -\frac{2}{5}$$
$$2^{2x+3} = 32$$	
$$3^{1-3x} = 81$$	

Worked example	Your turn
Solve:	Solve:
$$5 = 25^x$$	$$7 = 343^{x-2}$$
	$$x = \frac{7}{3}$$
$$3 = 27^x$$	
$$2 = 16^{x-1}$$	

Worked example	Your turn
Solve:	Solve:

$$5^x = \frac{1}{125}$$

$$7^x = \frac{1}{343}$$

$$x = -3$$

$$3^x = \frac{1}{9}$$

$$2^x = \frac{1}{16}$$

Worked example	Your turn
Solve:	Solve:

Worked example:

$$5^{-x} = \frac{1}{125}$$

$$3^{-x} = \frac{1}{9}$$

$$2^{-x} = \frac{1}{16}$$

Your turn:

$$7^{-x} = \frac{1}{343}$$

$$x = 3$$

Worked example	Your turn
Solve:	Solve:
$$5^{-x} = 125$$	$$7^{-x} = 343$$
	$$x = -3$$
$$3^{-x} = 9$$	
$$2^{-x} = 16$$	

Worked example	Your turn
Solve:	Solve:
$$125^x = \frac{1}{5}$$	$$343^x = \frac{1}{7}$$
	$$x = -\frac{1}{3}$$
$$9^x = \frac{1}{3}$$	
$$8^x = \frac{1}{2}$$	

Worked example	Your turn
Solve:	Solve:

Worked example:

$$2^{3x-2} = \frac{1}{64}$$

$$3^{2-5x} = \frac{1}{81}$$

Your turn:

$$5^{3-2x} = \frac{1}{25}$$

$$x = \frac{5}{2}$$

Worked example	Your turn
Solve:	Solve:

Worked example:

$$2^x = \sqrt[3]{16}$$

$$3^{x-2} = \sqrt{27}$$

Your turn:

$$5^{x+1} = \sqrt[4]{25}$$

$$x = -\frac{1}{2}$$

Worked example	Your turn
Solve:	Solve:

Worked example:

$$2^{3x-1} = \sqrt[4]{32}$$

$$3^{2-5x} = \sqrt{243}$$

Your turn:

$$5^{3-2x} = \sqrt[3]{25}$$

$$x = \frac{7}{6}$$

Worked example	Your turn
Solve:	Solve:
$$2^{3x-5} = \frac{16}{\sqrt[5]{64}}$$	$$3^{2x-5} = \frac{27}{\sqrt{243}}$$
	$$x = \frac{11}{4}$$

Worked example	Your turn
Solve: $$16 \times 2^{3x-1} = \frac{1}{\sqrt{32}}$$	Solve: $$27 \times 3^{5x-2} = \frac{1}{\sqrt[3]{9}}$$ $$x = -\frac{1}{3}$$

Worked example	Your turn
Solve:	Solve:
$$(16^x)^3 = \frac{1}{8}$$	$$(9^x)^5 = \frac{1}{27}$$
	$$x = -\frac{3}{10}$$

Worked example	Your turn
Solve:	Solve:
$$\sqrt{2} \times 8^{2x-5} = \frac{1}{16}$$	$$\sqrt{3} \times 9^{4x-3} = \frac{1}{27}$$
	$$x = \frac{5}{16}$$

Worked example	Your turn
Solve: $$\frac{16^{4x-3}}{8^{5-2x}} = 4$$	Solve: $$\frac{9^{3-5x}}{27^{4x-3}} = 81$$ $$x = \frac{1}{2}$$

Worked example	Your turn
Express y in terms of x, given: $$2^x \times 2^y = 8$$	Express y in terms of x, given: $$5^x \times 5^y = 125$$ $$y = 3 - x$$

$$3^x \times 3^y = \sqrt{27}$$

Worked example	Your turn
Express y in terms of x, given: $$2^{3x} \times 8^{5y} = \frac{1}{4}$$	Express y in terms of x, given: $$5^{3x} \times 25^{4y} = \frac{1}{\sqrt{125}}$$ $$y = -\frac{3}{8}x - \frac{3}{16}$$

Worked example	Your turn
Express y in terms of x, given: $$\frac{2^{3x}}{2^{5y}} = 2\sqrt{2}$$	Express y in terms of x, given: $$\frac{5^{x}}{5^{4y}} = 125\sqrt{5}$$ $$y = \frac{x}{4} - \frac{7}{8}$$

$$\frac{3^{4x}}{3^{2y}} = \frac{1}{27}$$

Worked example	Your turn
Express y in terms of x, given:	Express y in terms of x, given:
$$\frac{2^{3x}}{8^{5y}} = 2\sqrt{2}$$	$$\frac{5^x}{25^{4y}} = 125\sqrt{5}$$
	$$y = \frac{x}{8} - \frac{7}{16}$$

$$\frac{3^{4x}}{9^{2y}} = \frac{1}{27}$$

Worked example	Your turn
Solve $\left(\dfrac{1}{2}\right)^x = 8$	Solve $\left(\dfrac{1}{3}\right)^x = 9$
	$x = -2$
Solve $25^{3-4x} = \dfrac{1}{125}$	Solve $9^{4-3x} = \dfrac{1}{81}$
	$x = 2$

Worked example	Your turn
Solve $x^{\frac{1}{2}} = 5$	Solve $x^{\frac{1}{4}} = 2$
	$x = 16$
Solve $x^{\frac{1}{3}} = 6$	

Worked example	Your turn
Solve $x^{-2} = 25$	Solve $x^{-4} = 16$
	$x = \dfrac{1}{2}$
Solve $x^{-3} = 216$	

Worked example	Your turn
Given that $5^{-n} = 0.4$, find the value of $(5^3)^n$	Given that $3^{-n} = 0.2$, find the value of $(3^4)^n$
	625

Worked example	Your turn
Express $\dfrac{\left(3+\sqrt[3]{x}\right)^2}{x}$ as powers of x.	Express $\dfrac{\left(2+\sqrt{x}\right)^2}{x}$ as powers of x.
	$4x^{-1} + 4x^{-\frac{1}{2}} + 1$

Worked example	Your turn
Express $\dfrac{\left(x+\sqrt[3]{x}\right)^2}{4\sqrt[3]{x}}$ as powers of x.	Express $\dfrac{\left(x+\sqrt{x}\right)^2}{2\sqrt{x}}$ as powers of x.
	$\dfrac{1}{2}x^{\frac{3}{2}} + x + \dfrac{1}{2}x^{\frac{1}{2}}$

1.5) Surds

Worked example	Your turn
$\sqrt{x} \times \sqrt{x}$	$\sqrt[4]{z} \times \sqrt[4]{z} \times \sqrt[4]{z} \times \sqrt[4]{z}$
	z
$\sqrt[3]{y} \times \sqrt[3]{y} \times \sqrt[3]{y}$	

Worked example	Your turn
$2\sqrt{x} \times 3\sqrt{x}$	$2\sqrt[4]{z} \times 3\sqrt[4]{z} \times 4\sqrt[4]{z} \times \sqrt[4]{z}$
	$24z$
$\sqrt[3]{y} \times 3\sqrt[3]{y} \times 4\sqrt[3]{y}$	

Worked example	Your turn
Simplify:	Simplify:
$$\sqrt{72} - \sqrt{18}$$	$$\sqrt{75} - \sqrt{12}$$
	$3\sqrt{3}$
$$\sqrt{125} - \sqrt{45}$$	

Worked example	Your turn
Express in the form $a + b\sqrt{3}$: $$\left(5 + 3\sqrt{3}\right)^2$$	Express in the form $a + b\sqrt{5}$: $$\left(2 + 7\sqrt{5}\right)^2$$ $249 + 28\sqrt{5}$

Worked example	Your turn
Express in the form $a + b\sqrt{3}$: $$\left(2\sqrt{3} - 5\right)^2$$	Express in the form $a + b\sqrt{5}$: $$\left(3\sqrt{5} - 2\right)^2$$ $$49 - 12\sqrt{5}$$

Worked example	Your turn
Simplify:	Simplify:
$(\sqrt{7}+1)(\sqrt{7}-1)$	$(\sqrt{11}-3)(\sqrt{11}+3)$
	2
$(\sqrt{5}-2)(\sqrt{5}+2)$	

Worked example	Your turn
Simplify:	Simplify:
$$\left(7 + 4\sqrt{2}\right)\left(7 - 4\sqrt{2}\right)$$	$$\left(9 - 3\sqrt{5}\right)\left(9 + 3\sqrt{5}\right)$$
	36
$$\left(5 - 2\sqrt{3}\right)\left(5 + 2\sqrt{3}\right)$$	

Worked example	Your turn
Simplify: $$\left(7\sqrt{3}+4\sqrt{2}\right)\left(7\sqrt{3}-4\sqrt{2}\right)$$	Simplify: $$\left(11\sqrt{5}-3\sqrt{7}\right)\left(11\sqrt{5}+3\sqrt{7}\right)$$ $$542$$
$$\left(5\sqrt{7}-2\sqrt{3}\right)\left(5\sqrt{7}+2\sqrt{3}\right)$$	

Worked example	Your turn
Express in the form $a + b\sqrt{3}$: $(4 + \sqrt{3})(1 + 2\sqrt{3})$	Express in the form $a + b\sqrt{5}$: $(2 + \sqrt{5})(3 + 4\sqrt{5})$ $26 + 11\sqrt{5}$

Worked example	Your turn
Given that $a > 0$, show that $$\sqrt{5a}\left(\sqrt{20a} + a\sqrt{5a}\right)$$ Is always a multiple of 3	Given that $a > 0$, show that $$\sqrt{3a}\left(\sqrt{12a} + a\sqrt{3a}\right)$$ is always a multiple of 3
	Shown

Worked example	Your turn
Simplify	Simplify
$$\left(3 + \sqrt{2}\right)^2 - \left(3 - \sqrt{2}\right)^2$$	$$\left(2 + \sqrt{3}\right)^2 - \left(2 - \sqrt{3}\right)^2$$
	$$8\sqrt{3}$$
$$\left(4 + \sqrt{5}\right)^2 - \left(4 - \sqrt{5}\right)^2$$	

Worked example	Your turn
Find the value of a and b: $$\left(a - 3\sqrt{5}\right)^2 = b - 42\sqrt{5}$$	Express b and c in terms of a $$\left(a - 2\sqrt{3}\right)^2 = b - 20\sqrt{3}$$ $$a = 5, b = 37$$

Worked example	Your turn
Expand and simplify: $$(\sqrt{7} - 3)(\sqrt{7} + 2)(\sqrt{7} + 1)$$	Expand and simplify: $$(\sqrt{5} + 3)(\sqrt{5} - 2)(\sqrt{5} + 1)$$ 4

Worked example	Your turn
Expand and simplify: $(\sqrt{7} - 3)(\sqrt{7} + 2)(\sqrt{7} + 1)$	Expand and simplify: $(\sqrt{5} + 3)(\sqrt{5} - 2)(\sqrt{5} + 1)$ 4

1.6) Rationalising denominators

Worked example	Your turn
Rationalise the denominator:	Rationalise the denominator:
$$\frac{1}{\sqrt{2}}$$	$$\frac{4}{\sqrt{7}}$$
	$$\frac{4\sqrt{7}}{7}$$
$$\frac{2}{\sqrt{3}}$$	
$$\frac{3}{\sqrt{5}}$$	

Worked example	Your turn
Rationalise the denominator and simplify: $$\dfrac{4}{3\sqrt{2}}$$ $$\dfrac{6}{5\sqrt{3}}$$	Rationalise the denominator and simplify: $$\dfrac{10}{2\sqrt{7}}$$ $$\dfrac{5\sqrt{7}}{7}$$

Worked example	Your turn
Rationalise the denominator and simplify:	Rationalise the denominator and simplify:

Worked example:

$$\frac{2 - 7\sqrt{3}}{\sqrt{3}}$$

$$\frac{3 - 5\sqrt{11}}{\sqrt{11}}$$

Your turn:

$$\frac{9 - 3\sqrt{5}}{\sqrt{5}}$$

$$\frac{9\sqrt{5} - 15}{5}$$

Worked example	Your turn
Rationalise the denominator and simplify: $$\frac{3}{5 + \sqrt{2}}$$	Rationalise the denominator and simplify: $$\frac{18}{1 + \sqrt{7}}$$ $$-3 + 3\sqrt{7}$$
$$\frac{4}{1 + \sqrt{3}}$$	

Worked example	Your turn
Rationalise the denominator and simplify: $$\frac{3}{5 - \sqrt{2}}$$	Rationalise the denominator and simplify: $$\frac{18}{1 - \sqrt{7}}$$ $$\color{red}{-3 - 3\sqrt{7}}$$
$$\frac{4}{1 - \sqrt{3}}$$	

Worked example	Your turn
Rationalise the denominator and simplify: $$\frac{3}{\sqrt{2}-5}$$ $$\frac{4}{-1+\sqrt{3}}$$	Rationalise the denominator and simplify: $$\frac{18}{\sqrt{7}-1}$$ $$3+3\sqrt{7}$$

Worked example	Your turn
Rationalise the denominator and simplify: $$\frac{3 + \sqrt{2}}{\sqrt{2} - 5}$$	Rationalise the denominator and simplify: $$\frac{2 - \sqrt{7}}{\sqrt{7} - 1}$$ $$\frac{-5 + \sqrt{7}}{6}$$
$$\frac{5 - \sqrt{3}}{-1 + \sqrt{3}}$$	

Worked example	Your turn
Rationalise the denominator and simplify: $$\frac{2\sqrt{3} + 5}{\sqrt{3} - 2}$$	Rationalise the denominator and simplify: $$\frac{2\sqrt{2} + 1}{\sqrt{2} - 3}$$ $$-1 - \sqrt{2}$$

Worked example	Your turn
Rationalise the denominator and simplify:	Rationalise the denominator and simplify:
$$\dfrac{3\sqrt{3}-5}{2\sqrt{3}-7}$$	$$\dfrac{5\sqrt{5}-2}{2\sqrt{5}-3}$$
	$4+\sqrt{5}$

Worked example	Your turn
Rationalise the denominator and simplify: $$\frac{11 + 3\sqrt{2}}{3\sqrt{7} + 5}$$	Rationalise: $$\frac{7 + 2\sqrt{3}}{11\sqrt{5} + 2}$$ $$\frac{77\sqrt{5} + 22\sqrt{15} - 4\sqrt{3} - 14}{601}$$

Worked example	Your turn
Rationalise the denominator and simplify: $$\dfrac{\left(3+\sqrt{2}\right)\left(3-\sqrt{2}\right)}{\sqrt{7}}$$	Rationalise the denominator and simplify: $$\dfrac{\left(5+\sqrt{3}\right)\left(5-\sqrt{3}\right)}{\sqrt{22}}$$ $$\sqrt{22}$$
$$\dfrac{\left(7+\sqrt{5}\right)\left(7-\sqrt{5}\right)}{\sqrt{11}}$$	

Worked example	Your turn
Simplify, giving your answer in the form $a\sqrt{b}$: $$\frac{\sqrt{54}}{3} + \frac{12}{\sqrt{6}}$$	Simplify, giving your answer in the form $a\sqrt{b}$: $$\frac{\sqrt{63}}{3} + \frac{21}{\sqrt{7}}$$ $4\sqrt{7}$

Worked example	Your turn
Show that $\dfrac{1}{\frac{1}{\sqrt{3}}+1} = \dfrac{3}{2} - \dfrac{1}{2}\sqrt{3}$	Show that $\dfrac{1}{1+\frac{1}{\sqrt{2}}} = 2 - \sqrt{2}$

$$= \frac{\sqrt{2}}{\sqrt{2}+1}$$

$$= \frac{\sqrt{2}}{\sqrt{2}+1} \times \frac{\sqrt{2}-1}{\sqrt{2}-1}$$

$$= \frac{2-\sqrt{2}}{2+\sqrt{2}-\sqrt{2}-1}$$

$$= \frac{2-\sqrt{2}}{1}$$

$$= 2 - \sqrt{2}$$

Worked example	Your turn
A rectangle has an area of $(15 - 6\sqrt{3})\ cm^2$ and a width of $(2\sqrt{3} - 3)\ cm.$ Find the length of the rectangle in the form $a + b\sqrt{3}$	A rectangle has an area of $(2 + \sqrt{2})\ cm^2$ and a width of $(3\sqrt{2} - 4)\ cm.$ Find the length of the rectangle in the form $a + b\sqrt{2}$
	$7 + 5\sqrt{2}\ cm$

Worked example	Your turn
Solve $y\left(2 + \sqrt{3}\right) - \sqrt{3} = 4$	Solve $y\left(1 + \sqrt{2}\right) - \sqrt{2} = 3$
	$y = 2\sqrt{2} - 1$

Worked example	Your turn
Simplify: $$\frac{\sqrt{a+2}-\sqrt{a}}{\sqrt{a+2}+\sqrt{a}}$$	Simplify: $$\frac{\sqrt{a+1}-\sqrt{a}}{\sqrt{a+1}+\sqrt{a}}$$ $$2a+1-2\sqrt{a}\sqrt{a+1}$$

Worked example	Your turn
Solve: $$\frac{\sqrt[5]{8}}{\sqrt[4]{32}} = \sqrt[x]{2}$$	Solve: $$\frac{\sqrt[4]{9}}{\sqrt[5]{27}} = \sqrt[x]{3}$$ $$x = -10$$