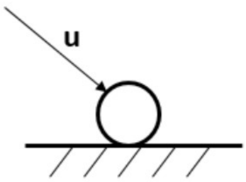
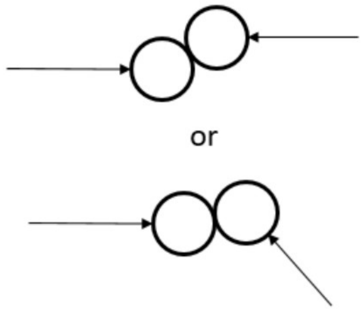
Elastic Collisions in Two Dimensions (Chapter 5)

Oblique impact with a fixed smooth surface

In this chapter we will consider:





What happens when a sphere hits a wall at an angle other than 90°?

What happens when two spheres that are not travelling along the same straight line collide?

The key to all the questions in this chapter is that **all** the spheres and **all** the surfaces are **ALWAYS SMOOTH.**

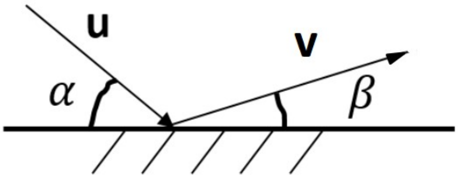
A smooth surface cannot apply a frictional force.
It can only apply a normal force.

The impulse is always normal to the surface, so momentum (and velocity) is **only changed perpendicular to the surface**.

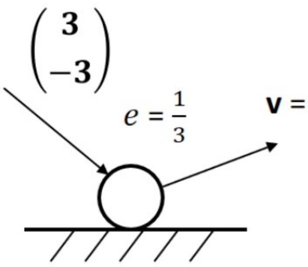
This means momentum (and velocity) **parallel to the surface remains unchanged**.

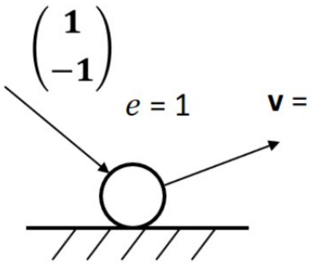
✏ The component of velocity **parallel** to the surfaces in contact is **unchanged**
✏ The component of velocity **perpendicular** to the surfaces in contact depends on **the coefficient of restitution (e).**

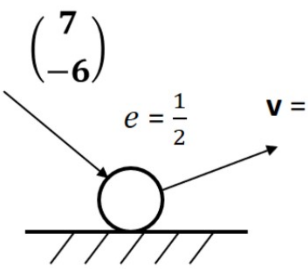
The theory

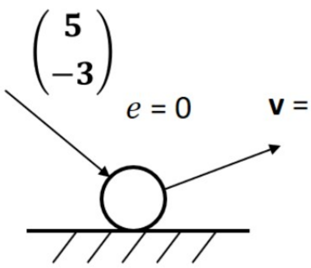


The angle of deflection is the total angle through which the path of the sphere changes.
With this diagram it is α + β









Example 1

A smooth sphere S is moving on a smooth horizontal plane with speed u when it collides with a smooth fixed vertical wall. At the instant of collision the direction of motion of S makes an angle of 60° with the wall. The coefficient of restitution between S and the wall is $\frac{1}{4}$. Find:

a the speed of S immediately after the collision

b the angle of deflection of S. *This does not depend on the initial speed. Let us check why!*

Tip: Many questions involve two equations and two unknowns which can be found in one step by:

- $(eqn\ 1)^2 + (eqn\ 2)^2$
- eqn 1 ÷ eqn 2

Ex 5A Q1-4

Example 2

A small smooth ball is falling vertically. The ball strikes a smooth plane which is inclined at an angle α to the horizontal, where $\tan \alpha = \frac{1}{2}$. Immediately before striking the plane the ball has speed $5\ \text{m s}^{-1}$. The coefficient of restitution between the ball and the plane is $\frac{1}{2}$

Find the speed of the ball immediately after the impact.

Ex 5A Q5-8

Example 3

A small smooth ball of mass 2 kg is moving in the xy-plane and collides with a smooth fixed vertical wall which contains the y-axis. The velocity of the ball just before impact is $(-6\mathbf{i} - 4\mathbf{j})\text{ m s}^{-1}$. The coefficient of restitution between the sphere and the wall is $\frac{1}{3}$. Find:

a the velocity of the ball immediately after the impact

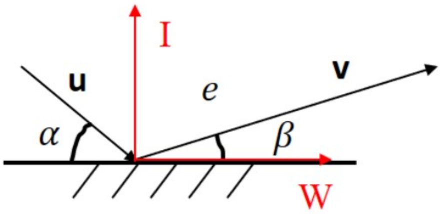
b the kinetic energy lost as a result of the impact

c the angle of deflection of the ball.

What knowledge can we use from vectors to help us?

Ex 5A Q9-10

Using the Scalar Product - when the wall is not in the 'i' or 'j' directions



Let's explore u.w and u.I

$-e\mathbf{u}.\mathbf{I} = \mathbf{v}.\mathbf{I}$
$\mathbf{u}.\mathbf{w} = \mathbf{v}.\mathbf{w}$

A smooth sphere S , of mass m, is moving with velocity $(2\boldsymbol{i} + 7\boldsymbol{j})ms^{-1}$ when it collides with a smooth fixed vertical wall.
After the collision the velocity of the sphere, S, is $(\boldsymbol{i} - 3\boldsymbol{j})ms^{-1}$

a) Find the impulse exerted by the wall on the ball.
b) Use the scalar product to find the coefficient of restitution between the sphere and the wall.

Q15, 16
Q11 - you'll need to describe the wall in vector form, and then find out the impulse. This one is harder! Ask if you need me to go through it

5. A small ball of mass 0.5 kg is moving on a smooth horizontal plane with velocity $(4\mathbf{i} - \mathbf{j})\,\text{m s}^{-1}$ when it strikes a fixed vertical wall. Immediately after the impact the velocity of the ball is $(2\mathbf{i} + 3\mathbf{j})\,\text{m s}^{-1}$. The ball is modelled as a particle and the wall is modelled as a smooth plane surface.

(a) Find the magnitude of the impulse of the wall on the ball in the impact.

(4)

(b) Find the loss in kinetic energy of the ball due to its impact with the wall.

(3)

(c) Find the coefficient of restitution between the ball and the wall.

(5)

(d) Verify that the component of the momentum of the ball, parallel to the line of intersection of the wall and the horizontal plane, is unchanged by the impact.

(2)

(e) State which modelling assumption ensures that the component of the momentum of the ball, parallel to the line of intersection of the wall and the horizontal plane, is unchanged by the impact.

(1)

11 A small smooth ball of mass 2 kg is moving in the xy-plane and collides with a smooth fixed vertical wall which contains the line $y = x$. The velocity of the ball just before impact is $(4\mathbf{i} + 2\mathbf{j})$ m s^{-1}. The coefficient of restitution between the sphere and the wall is $\frac{1}{3}$. Find:

a the velocity of the ball immediately after the impact

b the proportion of the original kinetic energy lost as a result of the impact

c the angle of deflection of the ball.

Successive oblique impacts

Sometimes you may be asked to consider two successive impacts.
1)Find the speed and direction of motion after the first impact.
2)Use angle properties to calculate the angle of approach for the second collision using the direction of motion after the first impact.
3)Look at the second impact starting by drawing a new diagram.

There are no new concepts needed to address this type of question.

Example 4

Two vertical walls meet at right angles. A smooth sphere slides across a smooth, horizontal floor, bouncing off each wall in turn. Just before the first impact the sphere is moving with speed 4 m s^{-1} at an angle of 30° to the wall. The coefficient of restitution between the sphere and both walls is $\frac{3}{4}$
Find:

a the direction of motion and speed of the sphere after the first collision

b the direction of motion and speed of the sphere after the second collision.

Example 5

Two cushions of a snooker table W_1 and W_2 meet at right angles. A snooker ball travels across the table and collides with W_1 and then W_2. The cushions are modelled as smooth.
Just before the first impact the ball is moving with speed $u \text{ m s}^{-1}$ at an angle of 20° to W_1.
The coefficients of restitution between the ball and the cushions W_1 and W_2 are $\frac{1}{2}$ and $\frac{2}{5}$ respectively.

a Find the percentage of the ball's original kinetic energy that is lost in the collisions.

b In reality the cushions may not be smooth. What effect will the model have had on the calculation of the percentage of kinetic energy remaining?

Example 6

Two smooth vertical walls stand on a smooth horizontal surface and intersect at an angle of 60°. A smooth sphere is projected across the surface with speed $1\ \text{m s}^{-1}$ at an angle of 20° to one of the walls and towards the intersection of the walls. The coefficient of restitution between the sphere and the walls is 0.4. Work out the speed and direction of motion of the sphere after:

a the first collision **b** the second collision

Ex 5B odd

Edexcel M4 June 2015 Q7a

Figure 4 represents the plan view of part of a smooth horizontal floor, where AB and BC are smooth vertical walls. The angle between AB and BC is 120°. A ball is projected along the floor towards AB with speed u m s–1 on a path at an angle of 60° to AB. The ball hits AB and then hits BC. The ball is modelled as a particle. The coefficient of restitution between the ball and each wall is $\frac{1}{2}$.

(a) Show that the speed of the ball immediately after it has hit AB is $\frac{\sqrt{7}}{4}$ u . (6)

The speed of the ball immediately after it has hit *BC* is *w* m s^{-1}.

(*b*) Find *w* in terms of *u*. (7)

Angle of approach with $BC = 19.1°$
$v\cos 19.1 = w\cos\phi$
$\frac{1}{2}v\sin 19.1 = w\sin\phi$
Form equation in v and ϕ
$w^2 = v^2\left(\frac{1}{4}\sin^2 19.1 + \cos^2 19.1\right)$
$0.634u$

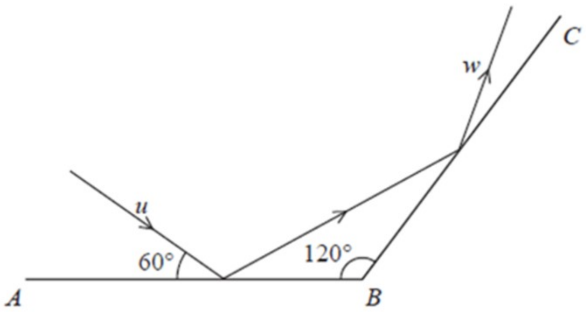


Figure 4

4.

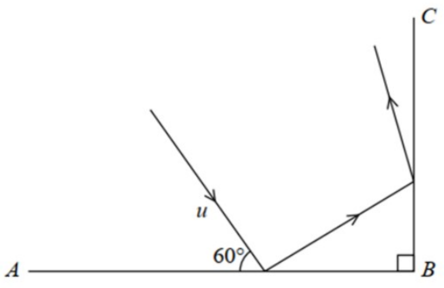


Figure 1

Figure 1 represents the plan view of part of a horizontal floor, where AB and BC are perpendicular vertical walls.

The floor and the walls are modelled as smooth.

A ball is projected along the floor towards AB with speed $u\,\text{m}\,\text{s}^{-1}$ on a path at an angle of 60° to AB. The ball hits AB and then hits BC.

The ball is modelled as a particle.

The coefficient of restitution between the ball and wall AB is $\frac{1}{\sqrt{3}}$

The coefficient of restitution between the ball and wall BC is $\sqrt{\frac{2}{5}}$

(a) Show that, using this model, the final kinetic energy of the ball is 35% of the initial kinetic energy of the ball.

(8)

(b) In reality the floor and the walls may not be smooth. What effect will the model have had on the calculation of the percentage of kinetic energy remaining?

(1)

5. [*In this question* **i** *and* **j** *are perpendicular unit vectors in a horizontal plane*]

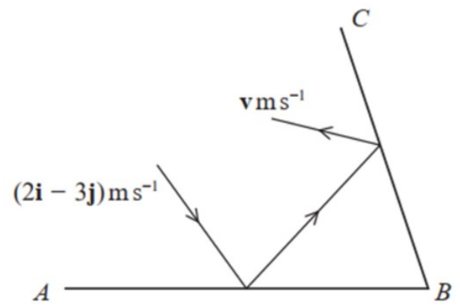


Figure 3

Figure 3 represents the plan view of part of a horizontal floor, where AB and BC represent fixed vertical walls. The direction of $\overrightarrow{AB}$ is in the direction of the vector **i** and the direction of $\overrightarrow{BC}$ is in the direction of the vector $(-\mathbf{i} + 3\mathbf{j})$.

A small ball is projected along the floor towards wall AB so that, immediately before hitting wall AB, the velocity of the ball is $(2\mathbf{i} - 3\mathbf{j})\,\text{m s}^{-1}$.

The ball hits wall AB and then hits wall BC.

The coefficient of restitution between the ball and wall AB is $\frac{1}{2}$

The coefficient of restitution between the ball and wall BC is $\frac{1}{3}$

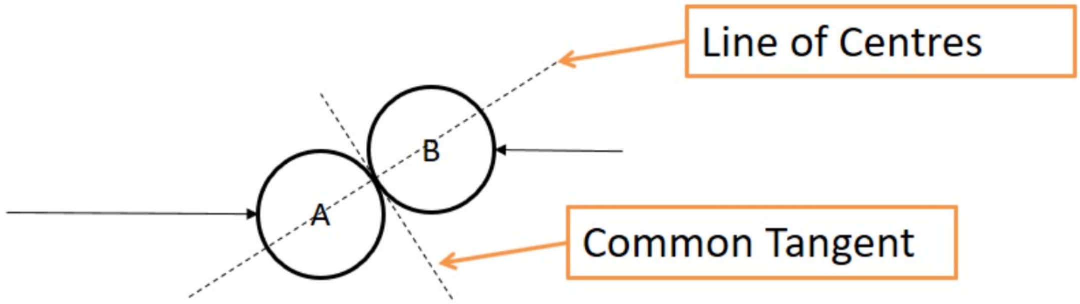
The velocity of the ball immediately after hitting wall BC is $\mathbf{v}\,\text{m s}^{-1}$.

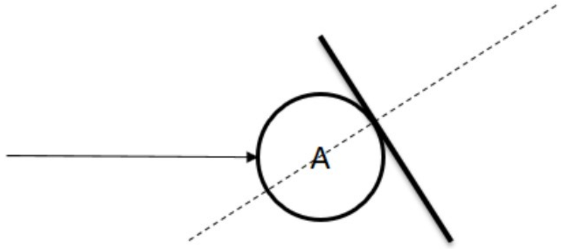
The floor and the walls are modelled as being smooth. The ball is modelled as a particle.

Show that $\mathbf{v} = \left(-\mathbf{i} + \frac{1}{2}\mathbf{j}\right)$.

(12)

Oblique Impact of Smooth Spheres





Key idea 1:
As with the smooth wall, the *component of velocity parallel to the common tangent* doesn't change.

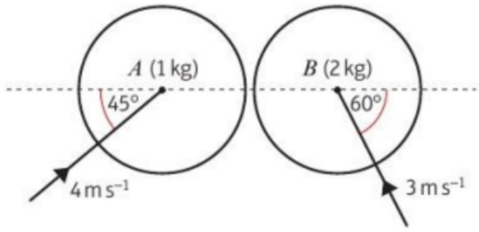
Key idea 2:
The *components of velocities parallel to the line of centres* are treated exactly like they were in Chapter 4 :

- Use PCLM
- Consider NLR

Example 7

A smooth sphere A, of mass 2 kg and moving with speed $6\,\text{m s}^{-1}$ collides obliquely with a smooth sphere B of mass 4 kg. Just before the impact B is stationary and the velocity of A makes an angle of 60° with the lines of centres of the two spheres. The coefficient of restitution between the spheres is $\frac{1}{4}$. Find the magnitudes and directions of the velocities of A and B immediately after the impact.

Example 8



A small smooth sphere A of mass 1 kg collides with a small smooth sphere B of mass 2 kg. Just before the impact A is moving with a speed of 4 m s^{-1} in a direction at 45° to the line of centres and B is moving with speed 3 m s^{-1} at 60° to the line of centres, as shown in the diagram. The coefficient of restitution between the spheres is $\frac{3}{4}$. Find:

a the kinetic energy lost in the impact

b the magnitude of the impulse exerted by A on B.

Example 9

A smooth sphere A of mass 5 kg is moving on a smooth horizontal surface with velocity $(2\mathbf{i} + 3\mathbf{j})\text{ m s}^{-1}$. Another smooth sphere B of mass 3 kg and the same radius as A is moving on the same surface with velocity $(4\mathbf{i} - 2\mathbf{j})\text{ m s}^{-1}$. The spheres collide when their line of centres is parallel to $\mathbf{j}$. The coefficient of restitution between the spheres is $\frac{3}{5}$. Find the velocities of both spheres after the impact.

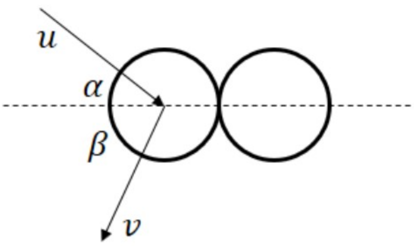
Example 10

Two small smooth spheres A and B have equal radii. The mass of A is $2m$ kg and the mass of B is $3m$ kg. The spheres are moving on a smooth horizontal plane and they collide. Immediately before the collision the velocity of A is $5\mathbf{j}$ m s^{-1} and the velocity of B is $(3\mathbf{i} - \mathbf{j})$ m s^{-1}. Immediately after the collision the velocity of A is $(3\mathbf{i} + 2\mathbf{j})$ m s^{-1}. Find:

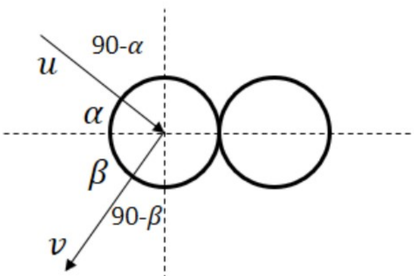
a the speed of B immediately after the collision

b a unit vector parallel to the line of centres of the spheres at the instant of the collision.

Angle of deflection – a common source of errors

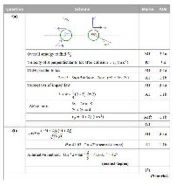


The angle of deflection is NOT $\alpha + \beta$.



The angle of deflection IS $(90 - \alpha) + (90 - \beta) = \mathbf{180 - \alpha - \beta}$.

6. [In this question **i** and **j** are perpendicular unit vectors in a horizontal plane.]



A smooth uniform sphere A has mass $2m$ kg and another smooth uniform sphere B, with the same radius as A, has mass $3m$ kg.

The spheres are moving on a smooth horizontal plane when they collide obliquely.

Immediately before the collision the velocity of A is $(3\mathbf{i} + 3\mathbf{j})\,\text{m s}^{-1}$ and the velocity of B is $(-5\mathbf{i} + 2\mathbf{j})\,\text{m s}^{-1}$.

At the instant of collision, the line joining the centres of the spheres is parallel to **i**.

The coefficient of restitution between the spheres is $\frac{1}{4}$

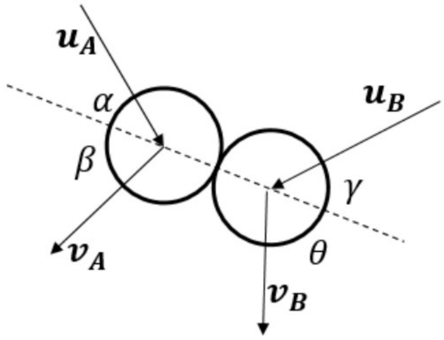
(a) Find the velocity of B immediately after the collision.

(7)

(b) Find, to the nearest degree, the size of the angle through which the direction of motion of B is deflected as a result of the collision.

(2)

Using the Scalar Product -
when the line of centres is not in the 'i' or 'j' directions



$$-e(\boldsymbol{u_A} - \boldsymbol{u_B}).\boldsymbol{I} = (\boldsymbol{v_A} - \boldsymbol{v_B}).\boldsymbol{I}$$

Note: Like the similar method introduced earlier for balls and walls, this is not explicitly covered in the textbook but is a good way to simplify some questions.
Note: There is only one qu in the textbook where this method is helpful: Ex 5C qu 14 but the answer is an impossible $e = -\frac{1}{7}$

Two small smooth spheres A and B have equal radii. The mass of A is $2m$kg and the mass of B is $20m$kg. The spheres are moving on a smooth horizontal plane and they collide. Immediately before the collision the velocity of A is $(2\boldsymbol{i} + \boldsymbol{j})ms^{-1}$ and B is stationary. Immediately after the collision the velocity of A is $2\boldsymbol{j}\ ms^{-1}$. Find:

a) The velocity of B after the collision
b) The coefficient of restitution between the two spheres

Edexcel M4 June 2012 Q1

A smooth uniform sphere S, of mass m, is moving on a smooth horizontal plane when it collides obliquely with another smooth uniform sphere T, of the same radius as S but of mass $2m$, which is at rest on the plane. Immediately before the collision the velocity of S makes an angle α, where $\tan \alpha = \frac{3}{4}$, with the line joining the centres of the spheres.

Immediately after the collision the speed of T is V. The coefficient of restitution between the two spheres is $\frac{3}{4}$.

(a) Find, in terms of V, the **speed** of S

(i) immediately before the collision,

(ii) immediately after the collision.

(9)

(b) Find the angle through which the direction of motion of S is deflected as a result of the collision.

(4)

Challenging Questions:
Review Exercise 2 qu 31, 36, 38
(Review Exercise 2 qu 19 – 39 are all good questions but these three combine several skills in an unusual way)