# Elastic Collisions in One Dimension (Chapter 4)

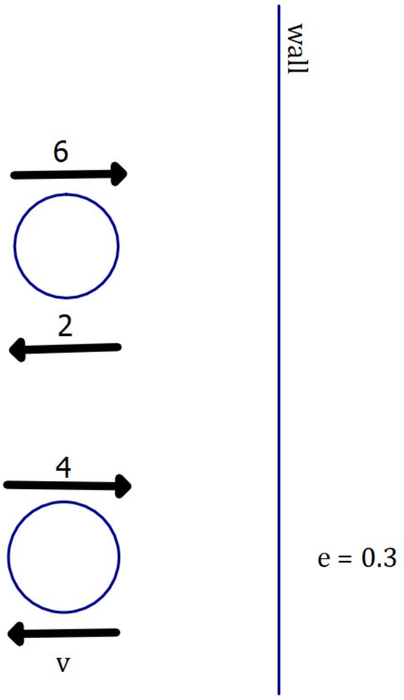
## Newton's Law of Restitution

$$0 \le e \le 1$$

$$e = \frac{\text{speed of separation}}{\text{speed of approach}}$$

**Inelastic, e = 0**
- plasticine
- particles immediately stop on collision and form one particle
- kinetic energy lost in collision

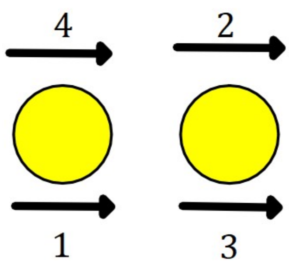
**Perfectly elastic, e = 1**
- table tennis ball (0.95)
- all kinetic energy is conserved




Ex 4B Q1-4

**2 balls colliding**

**Speed of separation and speed of approach**

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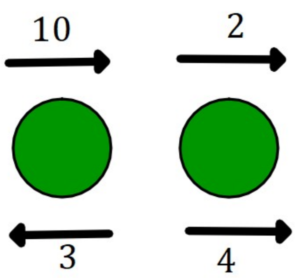


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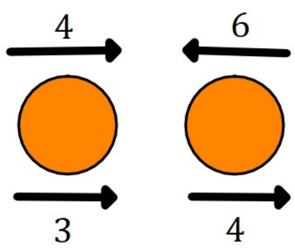


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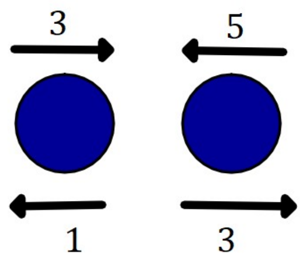


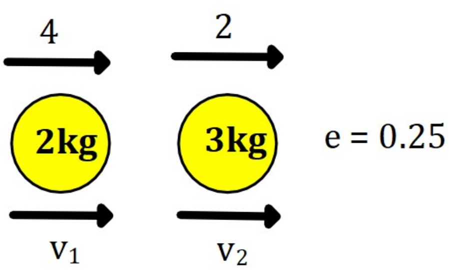

e =

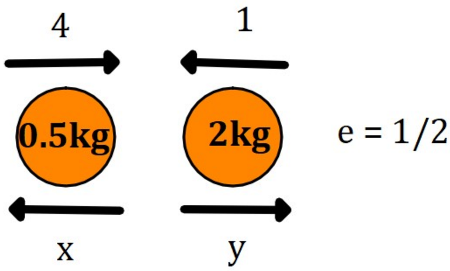
Newton's Law of Restitution

$$e = \frac{\text{speed of separation}}{\text{speed of approach}}$$

## 2 balls colliding

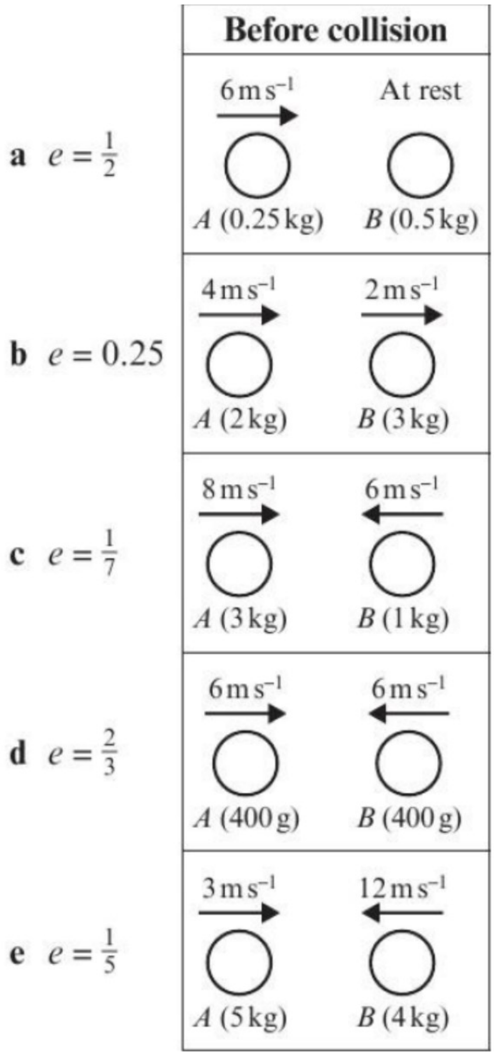
## Speed of separation and speed of approach




**Example 3**

Two particles $A$ and $B$ of masses 200 g and 400 g respectively are travelling in opposite directions towards each other on a smooth surface with speeds 5 m s$^{-1}$ and 4 m s$^{-1}$ respectively. They collide directly, and immediately after the collision have velocities $v_1$ m s$^{-1}$ and $v_2$ m s$^{-1}$ respectively, measured in the direction of motion of $A$ before the collision.
Given that the coefficient of restitution between $A$ and $B$ is $\frac{1}{2}$, find $v_1$ and $v_2$.




Find the velocities after the collision

- Use Newton's Law of Restitution
- Use PCLM

Ex 4A Q3-6

Two small spheres $P$ and $Q$ have mass $3m$ and $4m$ respectively. They are moving towards each other in opposite directions on a smooth horizontal plane. $P$ has speed $3u$ and $Q$ has speed $2u$ just before the impact. The coefficient of restitution between $P$ and $Q$ is $e$.

**a** Show that the speed of $Q$ after the collision is $\frac{u}{7}(15e + 1)$.

**b** Given that the direction of motion of $P$ is unchanged, find the range of possible values of $e$.

**c** Given that the magnitude of the impulse of $P$ on $Q$ is $\frac{80mu}{9}$, find the value of $e$.

Ex 4A Q7, 10, 8, 9

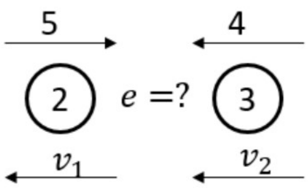
For 8 and 9, find what e is, and remember that $0 \leq e \leq 1$!

# Inequalities with collisions

Harder questions often ask for an inequality.
It is sometimes tricky to know where to start.
There are 4 common starting points:

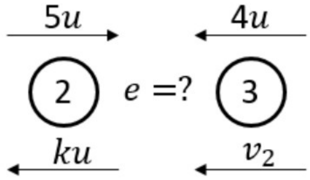
A. 'Direction of a particle is unchanged'
B. $0 \leq e \leq 1$
C. collision logic
D. 'the particles collide again' (considered later)

A.




Given that the direction of the 3kg particle is unchanged find the range of possible values of e.

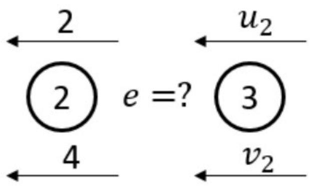
B.




Find the range of possible values of $k$.

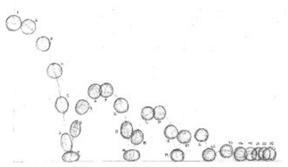
$0 \leq e \leq 1$

C.




Find the maximum value of $v_2$

A ball falls 22.5cm from rest onto a smooth horizontal plane.
It then rebounds to a height of 10cm.
Find the coefficient of restitution between the ball and the plane.
Give your answer to 2 significant figures.



Ex 4B
Q5-9

## Kinetic Energy

$\frac{1}{2}mv^2$ (consider each particle separately)

Loss in KE = Initial KE - Final KE

If e = 1, loss in KE = 0
If e = 0, loss in KE = initial KE

**Example 8**

Two spheres $A$ and $B$ have masses 3 kg and 5 kg respectively. $A$ and $B$ move towards each other in opposite directions along the same straight line on a smooth horizontal surface with speeds 3 m s$^{-1}$ and 2 m s$^{-1}$ respectively.

**a** Given the coefficient of restitution is $\frac{3}{5}$, find the velocities of the spheres after the collision.

**b** Find the loss of kinetic energy due to the impact.

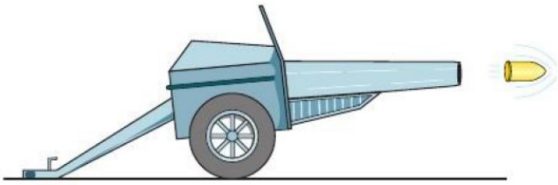
**Example 9**

A gun of mass 600 kg fires a shell of mass 12 kg horizontally with speed $200\,\text{m s}^{-1}$.

**a** Find the velocity of the gun after the shell has been fired.

**b** Find the total kinetic energy generated on firing.

**c** Show that the ratio of the energy of the gun to the energy of the shell is equal to the ratio of the speed of the gun to the speed of the shell after firing.



**Example 10**

Two particles $A$ and $B$, of masses 200 g and 300 g respectively, are connected by a light inextensible string. The particles are side by side at rest on a smooth floor and $A$ is projected with speed $6\,\text{m s}^{-1}$ directly away from $B$. When the string becomes taut, particle $B$ is jerked into motion and $A$ and $B$ then move with a common speed in the direction of projection of $A$. Find:

**a** the common speed of the particles after the string becomes taut

**b** the loss of total kinetic energy due to the jerk.

Ex 4C even questions

**Example 11** **Questions involving 3 balls**

Three spheres $A$, $B$ and $C$ have masses $m$, $2m$ and $3m$ respectively. The spheres move along the same straight line on a horizontal plane with $A$ following $B$, which is following $C$. Initially the speeds of $A$, $B$ and $C$ are $7\,\text{m}\,\text{s}^{-1}$, $3\,\text{m}\,\text{s}^{-1}$ and $1\,\text{m}\,\text{s}^{-1}$ respectively, in the direction $ABC$. Sphere $A$ collides with sphere $B$ and then sphere $B$ collides with sphere $C$. The coefficient of restitution between $A$ and $B$ is $\frac{1}{2}$ and the coefficient of restitution between $B$ and $C$ is $\frac{1}{4}$

**a** Find the velocities of the three spheres after the second collision.

**b** Explain how you can predict that there will be a further collision between $A$ and $B$.

## Example 12 Questions involving several collisions

A uniform smooth sphere $P$ of mass $3m$ is moving in a straight line with speed $u$ on a smooth horizontal table. Another uniform smooth sphere $Q$ of mass $m$ is moving with speed $2u$ in the same straight line as $P$, but in the opposite direction. The sphere $P$ collides with the sphere $Q$ directly. The velocities of $P$ and $Q$ after the collision are $v$ and $w$ respectively, measured in the direction of motion of $P$ before the collision. The coefficient of restitution between $P$ and $Q$ is $e$.

**a** Find expressions for $v$ and $w$ in terms of $u$ and $e$.

**b** Show that, if the direction of motion of $P$ is changed by the collision, then $e > \frac{1}{3}$

Following the collision with $P$, the sphere $Q$ then collides with and rebounds from a vertical wall, which is perpendicular to the direction of motion of $Q$. The coefficient of restitution between $Q$ and the wall is $e'$.

**c** Given that $e = \frac{5}{9}$ and that $P$ and $Q$ collide again in the subsequent motion, show that $e' > \frac{1}{9}$

**20** A particle $A$ of mass $2m$, moving with speed $2u$ in a straight line on a smooth horizontal table, collides with a particle $B$ of mass $3m$, moving with speed $u$ in the same direction as $A$. The coefficient of restitution between $A$ and $B$ is $e$.

**a** Show that the speed of $B$ after the collision is

$\frac{1}{5}u(7 + 2e)$.

**b** Find the speed of $A$ after the collision, in terms of $u$ and $e$.

The speed of $A$ after the collision is $\frac{11}{10}u$.

**c** Show that $e = \frac{1}{2}$.

At the instant of collision, $A$ and $B$ are at a distance $d$ from a vertical barrier fixed to the surface at right-angles to their direction of motion. Given that $B$ hits the barrier, and that the coefficient of restitution between $B$ and the barrier is $\frac{11}{16}$,

**d** find the distance of $A$ from the barrier at the instant that $B$ hits the barrier,

**e** show that, after $B$ rebounds from the barrier, it collides with $A$ again at a distance $\frac{5}{32}d$ from the barrier. E

**11** A mass of 2 kg moving at 35 m $s^{-1}$ catches up and collides with a mass of 10 kg moving in the same direction at 20 m $s^{-1}$. Five seconds after the impact the 10 kg mass encounters a fixed barrier which reduces it to rest. Assuming the coefficient of restitution between the masses is $\frac{3}{5}$, find the further time that will elapse before the 2 kg mass strikes the 10 kg mass again. You may assume that the masses are moving on a smooth surface and have constant velocity between collisions.

Example 13

## Questions involving multiple vertical bounces

A tennis ball, which may be modelled as a particle, is dropped from rest at a height of 90 cm onto a smooth horizontal plane. The coefficient of restitution between the ball and the plane is 0.5. Assume that there is no air resistance and that the ball falls under gravity and hits the plane at right angles.

**a** Find the height to which the ball rebounds after the first bounce.

**b** Find the height to which the ball rebounds after the second bounce.

**c** Find the total distance travelled by the ball before it comes to rest, according to this model.

**d** Criticise this model with respect to the motion of the ball as it continues to bounce.

Use the formula for the sum of an infinite geometric series:
$S = \frac{a}{1 - r}$