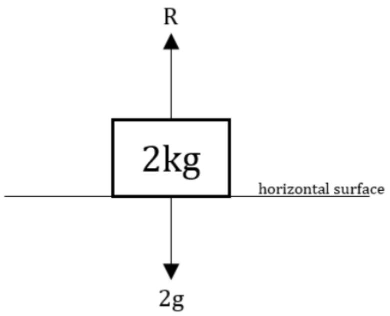
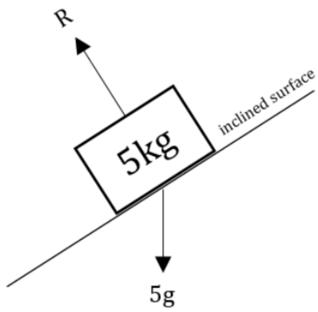
Mechanics essentials

weight = mass × g (where g is the acceleration due to gravity, $g = 9.8ms^{-2}$
$W = mg$
Weight acts vertically downwards (obviously)

The normal reaction (sometimes called the contact force) is the force which acts on a box/particle from the surface that it is on.
It is called a **normal** reaction because it acts normal (perpendicular) to the surface.
It is called a normal **reaction** because it has reacted to the forces in the opposing direction.
For example, when you are sat on a chair, your weight acts down, but the chair (surface) has a reaction force upwards which stops you falling to the floor. This is the normal reaction.
We use the letter R for the normal reaction.

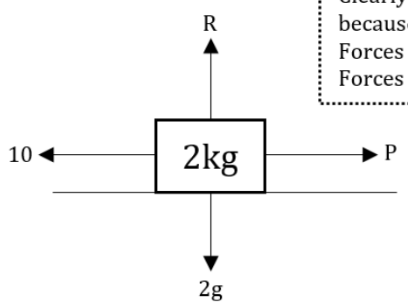




Note that the weight acts vertically downwards, but the normal reaction is perpendicular to the slope

Newton's First Law
"An object will remain at rest or will continue to move with constant velocity unless acted upon by an external force"
Essentially, this means that something will not move, or move with no acceleration if there is no overall resultant force. It means that all the forces are balanced.
We call this **equilibrium** (think of the word 'equal')

Given that this particle is rest, work out the value of P and R.
Clearly, R = 2g and P = 10, because it is in equilibrium.
Forces left = forces right.
Forces up = forces down



<u>Newton's Second Law</u>

"An object will accelerate if there is an overall resultant force on the object. The acceleration is proportional to this force, and inversely proportional to its mass."

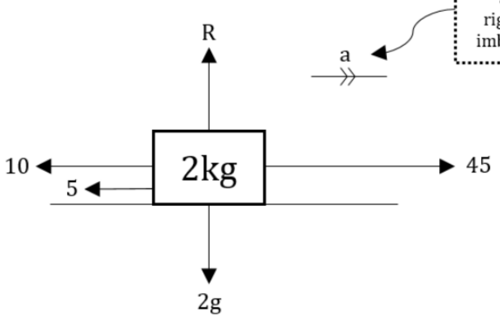
In other words

F = ma

Where **F** = resultant force, **m** = mass, **a** = acceleration

The resultant force is found by finding the difference between the forces in one direction, and the forces in the opposing direction. This tells you the overall force in one direction.

It is clearly going to accelerate to the right as there is an imbalance of forces



Notice how we find the resultant force by doing the forces to the right minus the forces to the left

Work out the acceleration for the particle.

Using F = ma

45 – 10 – 5 = 2a

30 = 2a

15 = a

So the particle will accelerate at $15ms^{-2}$ to the right

Maximum friction = coefficient of friction (μ) × normal reaction (R)

Maximum $F_r = \mu R$

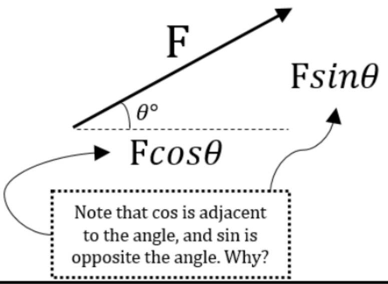
Note, the coefficient of friction is always greater than 0 and usually less than about 1.5.
The coefficient of friction is specific for the particle and the surface it is on.
A box on sandpaper would have a high coefficient of friction.
A box on ice would have a low coefficient of friction.
A box on a smooth surface would have a coefficient of friction = 0 (i.e. there is no friction)

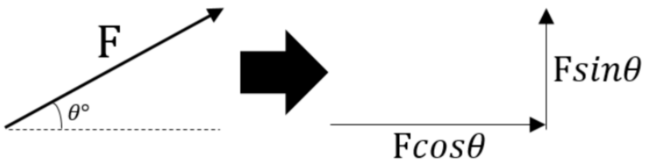
Friction acts in the **opposite direction to its motion** (obviously)
Friction can be less than this maximum value **if the particle is not moving**.

Forces at angles

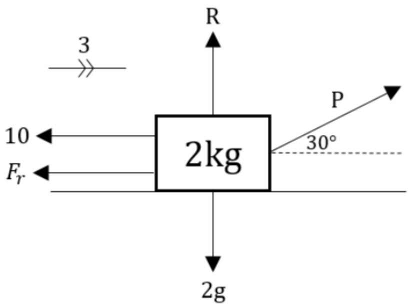
To consider forces at angles, and boxes on slopes, we have to simplify the situation.
We do this by resolving forces into perpendicular components, and then dealing with the up/down forces, and the left/right forces separately.

To separate a force, given its angle:





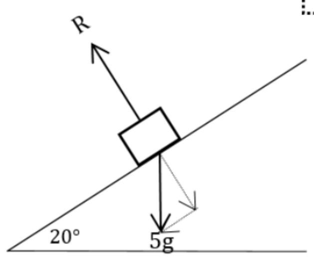
Hence a force of magnitude F at angle θ above the horizontal,
is equivalent to a force of magnitude **F$cos\theta$ to the right**, and **F$sin\theta$ up.**



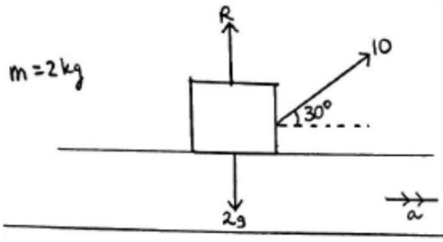
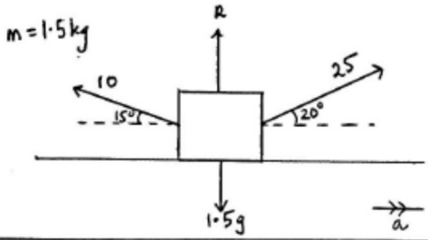
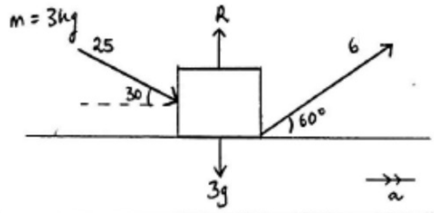
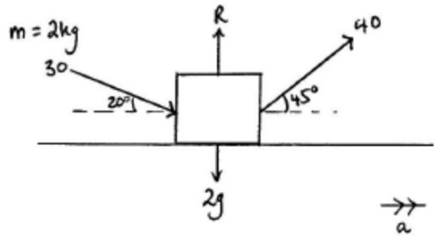
Given that the coefficient of friction is 0.5, work out the value of the pulling force, P.

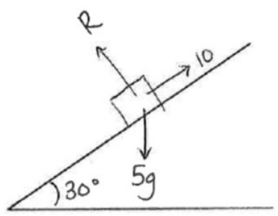
Working with slopes

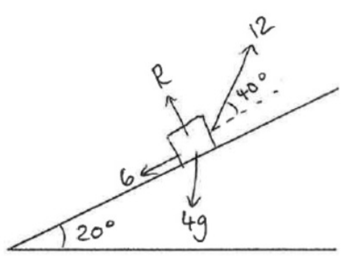
To consider forces on slopes, we resolve forces so that they are all either parallel to the slope, or perpendicular to the slope. Then we use the same strategies as before.

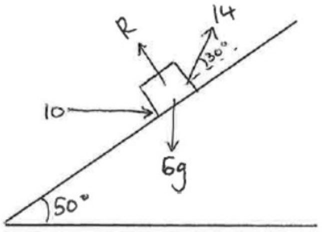


Given that the coefficient of friction is 0.1, work out the acceleration of the box down the slope.

	New Diagram	Find the value of R and a
		
		
		
		







Work

If an object has ***energy,*** it can do ***work***.
Work transfers energy from one place or form to another.
Work is done by a force when it moves an object.

✎ Work (a bit like energy) = Force × distance

Force and distance must be in the same direction

Units:
Force is in Newtons (N) and distance in meters (m)
... **Work** is in **Joules (J)**

A horizontal force of 8N moves a box 5m across a horizontal floor. Calculate the work done by the force.

A bricklayer raises a load of bricks of total mass 30kg at a constant speed by attaching a cable to the bricks. Assuming the cable is vertical, calculate the work done when the bricks are raised a distance of 7m.

Note: Work done against gravity is always:
weight (mg) × distance moved in direction of weight (h)
= mgh

Ex 2A Q1-8

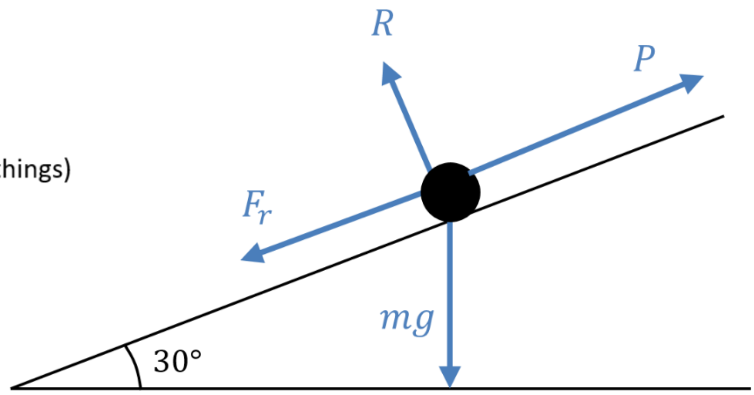
A box of mass 3kg is pushed across a rough horizontal floor.
The box moves at 3ms^{-1} and the coefficient of friction between the box and the floor is 0.45. Calculate the work done in 2 seconds.

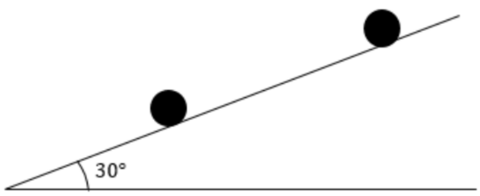
A boy of mass 40kg slides 3m down a slide, which is inclined at 25° to the horizontal. Modelling the slide as a smooth slope, calculate the work done by gravity.

Q9-13

What is the energy put into the system?

What does this energy get 'spent' on? (three things)





Note: Work done by P = Work done against friction + Work done against gravity + Kinetic energy gained.

A package of mass 2kg is pulled at a constant speed up a rough plane which is inclined at 30° to the horizontal. The coefficient of friction between the package and the surface is 0.35. The package is pulled 12m up a line of greatest slope of the plane. Calculate:

a) The work done against gravity
b) The work done against friction
c) The total work done by the pulling force.

A rough surface is inclined at 25° to the horizontal.
A box of mass 3kg is pulled at a constant speed up the surface by a force T acting along a line of greatest slope.
The coefficient of friction between the particle and the surface is 0.4.
Modelling the box as a particle, calculate the work done by T when the particle travels 8m up the slope.

A rough surface is inclined at an angle arcsin$\left(\frac{3}{5}\right)$ to the horizontal.
A particle of mass 2kg is pulled 3m at a constant speed up the surface by a force acting along a line of greatest slope.
The only resistances to motion are those due to friction and gravity.
The work done by the force is 50J.
Calculate the coefficient of friction between the particle and the surface.

Q14-17

Energy

K.E. = $\frac{1}{2}mv^2$

Mass	Velocity	Kinetic Energy
10 kg	5 m/s	
2 tonnes	3 ms^{-1}	
4 kg	(3i – 4j) ms^{-1}	
20 kg	(-5i + 12j) m/s	

G.P.E. = mgh

Δ G.P.E. = work done against gravity = $mg(h_2 - h_1)$, if h_1=0 this gives us G.P.E. =mgh

Note: choose a 'zero level' of potential energy before calculating a particles gravitational potential energy

Ex 2B Q2, 3, 4, 5

Combining F=ma and $v^2 = u^2 + 2as$

A box of mass 1.5kg is pulled across a smooth horizontal surface by a horizontal force. The initial speed of the box is u ms^{-1} and its final speed is $3\mathrm{ms}^{-1}$. The work done by the force is 1.8J. Calculate the value of u.

A van of mass 2000kg starts from rest at some traffic lights. After travelling 400m the van's speed is $12ms^{-1}$. A constant resistance of 500N acts on the van. Calculate the driving force, which can be assumed to be constant.

A parcel of mass 3kg is pulled 10m up a plane inclined at an angle $\theta°$ to the horizontal, where $\tan\theta = \frac{3}{4}$. Assuming that the parcel moves up the line of greatest slope of the plane

(a) Calculate the potential energy gained by the parcel.

(b) What would the speed of the parcel be if the gain in gravitational potential energy was all transferred into kinetic energy.

Ex 2B Q7-11

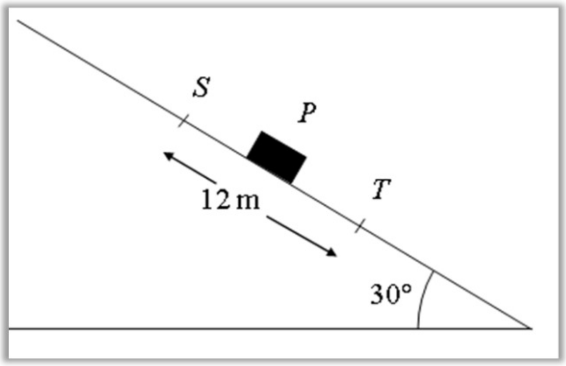
A small package P is modelled as a particle of mass 0.6 kg. The package slides down a rough plane from a point S to a point T, where $ST = 12$ m. The plane is inclined at an angle of 30° to the horizontal and ST is a line of greatest slope of the plane, as shown in Figure 3. The speed of P at S is 10 m s^{-1} and the speed of P at T is 9 m s^{-1}.

(*a*) Calculate the total loss of energy of P in moving from S to T,

(4)

(*b*) Given that the work done against friction by P is equal to total loss of energy of P in moving from S to T, calculate the coefficient of friction between P and the plane.

(5)



(a) KE lost is $\frac{1}{2} \times 0.6 \times (10^2 - 9^2)$ $(= 5.7\ J)$

PE lost is $0.6 \times 9.8 \times 12 \sin 30°$ $(= 35.28\ J)$

Total loss in energy is 41.0 (J)

(b) $R = 0.6 \times 9.8 \times \cos 30°$ $(= 5.09)$

WE $40.98 = \mu \times 0.6 \times 9.8 \times \cos 30° \times 12$

$\mu = 0.67$ or 0.671

The Law of Conservation of Energy (expanded)

Initial Energy = Final Energy

work done by engine +
initial GPE + initial KE = final GPE + final KE + w.d. against friction

Potential energy released/energy going in	=	Potential energy stored/energy going out

Consider the energy it has at the beginning - I tend to think of this as the energy it has in the bank (a bit like money).

Some of this energy is 'spent' in various ways - it is either spent and converted into another type of energy - or it is spent on having to overcome friction/resistance. Some energy is not spent, but is instead increased (e.g. the KE may increase if it gets faster because GPE is converted to KE)

If there is a force/engine doing work, then there is more energy 'in the bank' to be converted. This is why it is on the LHS of the equation.

A box of mass kg is projected from point A across a rough horizontal floor with speed $4ms^{-1}$. The box moves in a straight line across the floor and comes to rest at point B. The coefficient of friction between the box and the floor is 0.5

a) calculate the kinetic energy lost by the box

b) write down the work done against friction

c) calculate the distance AB

Ex 2C Q2, 4, 5, 6

A smooth plane is inclined at 30° to the horizontal. A particle of mass 0.5kg slides down a line of greatest slope of the plane. The particle starts from rest at point A and passes point B with a speed $6ms^{-1}$. Find the distance AB.

A particle of mass 2kg is projected with speed $8ms^{-1}$ up a line of greatest slope of a rough plane inclined at 45° to the horizontal. The coefficient of friction between the particle and the plane is 0.4. Calculate the distance the particle travels up the plane before coming to instantaneous rest.

A skier moving downhill passes point A on a ski run at $6ms^{-1}$. After descending 50m vertically the run begins to ascend. When the skier has ascended 25m to point B her speed is $4ms^{-1}$. The skier and her skis have a combined mass of 55kg. The total distance she travels from A to B is 1400m. The non-gravitational resistances to motion are constant and have a total magnitude of 12N.

Calculate the work done by the skier.

Ex 2C Q10-19

2.

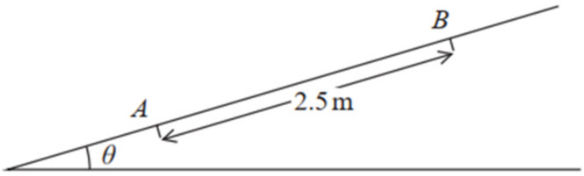


Figure 1

Figure 1 shows a ramp inclined at an angle θ to the horizontal, where $\sin\theta = \frac{2}{7}$

A parcel of mass 4 kg is projected, with speed $5\,\text{m}\,\text{s}^{-1}$, from a point A on the ramp.
The parcel moves up a line of greatest slope of the ramp and first comes to instantaneous rest at the point B, where $AB = 2.5$ m.
The parcel is modelled as a particle.

The total resistance to the motion of the parcel from non-gravitational forces is modelled as a constant force of magnitude R newtons.

(a) Use the work-energy principle to show that $R = 8.8$

(4)

After coming to instantaneous rest at B, the parcel slides back down the ramp. The total resistance to the motion of the particle is modelled as a constant force of magnitude 8.8 N.

(b) Find the speed of the parcel at the instant it returns to A.

(3)

(c) Suggest two improvements that could be made to the model.

(2)

s

Edexcel M2 June 2007 Q4

Two particles A and B, of mass m and $2m$ respectively, are attached to the ends of a light inextensible string. The particle A lies on a rough plane inclined at an angle α to the horizontal, where $\tan \alpha = \frac{3}{4}$. The string passes over a small light smooth pulley P fixed at the top of the plane. The particle B hangs freely below P, as shown in Figure 2. The particles are released from rest with the string taut and the section of the string from A to P parallel to a line of greatest slope of the plane. The coefficient of friction between A and the plane is $\frac{5}{8}$. When each particle has moved a distance h, B has not reached the ground and A has not reached P.

(*a*) Find an expression for the potential energy lost by the system when each particle has moved a distance h.

(2)

When each particle has moved a distance h, they are moving with speed v. Using the work-energy principle,

(*b*) find an expression for v^2, giving your answer in the form kgh, where k is a number.

(5)

Extension: If B starts *s* meters above the ground Find an expression, in terms of for the total distance travelled by A before it first comes to rest.

4. (a)	PE lost $= 2mgh - mgh \sin\alpha$ $(= 7mgh/5)$	M1 A1	(2)
(b)	Normal reaction $R = mg\cos\alpha$ $(= 4mg/5)$	B1	
	Work-energy: $\frac{1}{2}mv^2 + \frac{1}{2}.2mv^2 = \frac{7mgh}{5} - \frac{5}{8}.\frac{4mg}{5}h$	M1 A2,1,0	
	$\Rightarrow \frac{3}{2}mv^2 = \frac{9mgh}{10} \Rightarrow v^2 = \frac{3}{5}gh$	A1	(5)

Answer to Extension:

Let extra distance $= x$

$\Rightarrow \frac{1}{2}mv^2 = mgh + F_r x$

$\Rightarrow \frac{1}{2}m\frac{3}{5}gs = mgx\frac{3}{5} + \frac{5}{8}mg\frac{4}{5}x$

$\Rightarrow \frac{3}{10}s = \frac{11}{10}x$

$\Rightarrow x = \frac{3}{11}$

$\Rightarrow$ *Total distance* $= \frac{14s}{11}$

Power

Power is the **rate of doing work** or **<u>energy input per unit of time</u>**

Using the second definition:

$$Power = \frac{energy}{time} = \frac{Force \times distance}{time} = Force \times velocity = \boldsymbol{Fv}$$

Units: Power is measured in Watts (W).
A Watt is equivalent to a Js^{-1} or kgm^2s^{-3}.

A van of mass 1250kg is travelling along a horizontal road. The van's engine is working at 24kW. The constant resistance to motion has a magnitude of 600N. Calculate :

a) the acceleration of the van when it is travelling at $6ms^{-1}$

b) the maximum speed of the van.

A car of mass 1100kg is travelling at a constant speed of $15ms^{-1}$ along a straight road which is inclined at 7° to the horizontal. The engine is working at a rate of 24kW.

a) Calculate the magnitude of the non-gravitational resistance to motion.

The rate of working of the engine is now increased to 28kW.
Assuming the resistances to motion are unchanged,
b) Calculate the initial acceleration of the car.

A car of mass 2600kg is travelling in a straight line. At the instant when the speed of the car is $v \text{ ms}^{-1}$, the total resistances to motion are modelled as a variable force of magnitude $(800 + 5v^2)N$. The care has a cruise control feature which adjusts the power generated by the engine to maintain a constant speed of 18 ms^{-1}.
Find the power generated by the engine when:
a) The car is travelling on a horizontal road
b) The car is travelling up a road that is inclined at an angle 4° to the horizontal.

The rate of working of the engine is now increased to 28kW.
Assuming the resistances to motion are unchanged,
b) Calculate the initial acceleration of the car.

Ex 2D Q10-19

Side Note: Maximum speed is when acceleration is 0 - i.e. in equilibrium

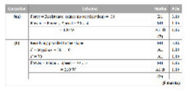
Edexcel M2 June 2003 Q6

A girl and her bicycle have a combined mass of 64 kg. She cycles up a straight stretch of road which is inclined at an angle α to the horizontal, where $\sin\alpha = \frac{1}{14}$. She cycles at a constant speed of 5 m s^{-1}. When she is cycling at this speed, the resistance to motion from non-gravitational forces has magnitude 20 N.

(*a*) Find the rate at which the cyclist is working.

(4)

8. [*In this question use* $g = 10\,\text{m}\,\text{s}^{-2}$]



A jogger of mass 60 kg runs along a straight horizontal road at a constant speed of $4\,\text{m}\,\text{s}^{-1}$. The total resistance to the motion of the jogger is modelled as a constant force of magnitude 30 N.

(a) Find the rate at which the jogger is working.

(3)

The jogger now comes to a hill which is inclined to the horizontal at an angle α, where $\sin\alpha = \frac{1}{15}$. Because of the hill, the jogger reduces her speed to $3\,\text{m}\,\text{s}^{-1}$ and maintains this constant speed as she runs up the hill. The total resistance to the motion of the jogger from non-gravitational forces continues to be modelled as a constant force of magnitude 30 N.

(b) Find the rate at which she has to work in order to run up the hill at $3\,\text{m}\,\text{s}^{-1}$.

(5)