CP2 Chapter 6

Hyperbolic Functions

Course Structure

1. Definition of hyperbolic functions and their sketches.
2. Inverse hyperbolic functions.
3. Hyperbolic Identities and Solving Equations
4. Differentiation
5. Integration



Conic Sections

In mathematics there are a number of different **families of curves**. Each of these have different properties and their equations have different forms.

It is possible to obtain these different types of curves by **slicing a cone**, hence “conic sections”.

The axis of the parabola is parallel to the side of the cone.

For interest:



What’s the point of hyperbolic functions?

Hyperbolic functions often result from differential equations (e.g. in mechanics), and we’ll see later in this module how we can use these functions in calculus.

For example, we can consider forces acting on each point on a hanging piece of string.

Solving the relevant differential equations, we end up with coshx.

Equations for Hyperbolic Functions

**Hyperbolic sine:**

**Hyperbolic cosine:**

**Hyperbolic tangent:**

**Hyperbolic secant:**

**Hyperbolic cosecant:**

**Hyperbolic cotangent:**

Examples:

1. Calculate (using both your button and using the formula)
2. Write in terms of :
3. Find the exact value of:

1. Solve

Sketching Hyperbolic Functions



* is the average of and :



* is the average of and :



*

Test Your Understanding

Sketch the graph of





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Inverse Hyperbolic Functions

Each hyperbolic function has an inverse.





Expressing Inverse Hyperbolic Functions in terms of

Given that hyperbolic functions can be written in terms of inverse hyperbolic can be expressed in terms of .

Example:

Prove that

Test Your Understanding

Prove that

Summary so far:



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Hyperbolic Identities

Use the definitions of and to find:

Also:

Hence:

Osborn’s Rule

We can get these identities from the normal sin/cos ones using Osborn’s rule:

Osborn’s Rule:

1. Replace and
2. **Negate** any explicit or implied **product of two sines**.

Solving Equations

To solve equations either use hyperbolic identities or basic definitions of hyperbolic functions.

Examples

1. Solve for all real
2. Solve for all real

Recap: If , find

1. If , find the exact value of:

a)

b)

c)

Test Your Understanding







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Differentiating hyperbolic functions

Example

Prove that

Test Your Understanding



Inverse Hyperbolic Functions



Proof of

Examples

1. Find
2. Given that prove that

Test Your Understanding





Using Maclaurin expansions for approximations

Textbook Example

(a) Show that **[We did this earlier]**

(b) Find the first two non-zero terms of the series expansion of .

The general form for the series expansion of is given by

(c) Find, in simplest terms, the coefficient of .

(d) Use your approximation up to and including the term in to find an approximate value for .

(e) Calculate the percentage error in using this approximation.

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Standard Integrals



Standard Integral Examples

Not Quite so Standard Examples

Using Identities

What now??

Sometimes there are techniques which work on non-hyperbolic trig functions but doesn’t work on hyperbolic ones. Just first replace any hyperbolic functions with their definition.

Examples

1. Find
2. Find

Dealing with , , ….

We can use substitution to deal with this style of integration.

* Consider . What substitution might we use?
* Consider. What substitution might we use?

Example

1. Show that
2. Show that
3. Show that .

Test Your Understanding



2)Using a hyperbolic substitution, evaluate

Integrating by Completing the Square

By completing the square, we can then use one of the standard results.

Examples

1. Determine
2. Determine

Test Your Understanding



Ex 6E and Mixed Ex 6