Core Pure 2

Volumes of Revolution

<u>Chapter Overview</u>

1: Revolving around the x-axis.

2: Revolving around the y-axis.

3: Volumes of revolution with parametric curves.

4: Modelling

5 **Further calculus**	5.1	**Derive formulae for and calculate volumes of revolution.**	**Both** $\pi\int y^2\,dx$ **and** $\pi\int x^2\,dy$ **are required.** Students should be able to find a volume of revolution given either Cartesian equations or parametric equations.

This chapter involves volumes of revolution but with trickier integration than in CP1.

Revolving around the x-axis

Recap: When revolving around the x-axis, $V = \pi \int_b^a y^2 \, dx$

Example

The region R is bounded by the curve with equation $y = \sin 2x$, the x-axis and $x = \frac{\pi}{2}$. Find the volume of the solid formed when region R is rotated through 2π radians about the x-axis.

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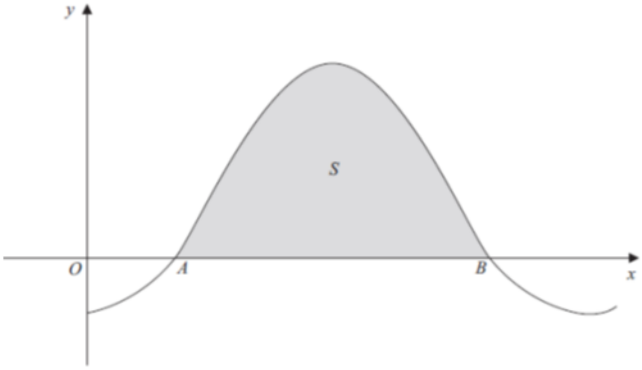


Figure 3 shows a sketch of part of the curve with equation $y = 1 - 2\cos x$, where x is measured in radians. The curve crosses the x-axis at the point A and at the point B.

(*a*) Find, in terms of π, the x coordinate of the point A and the x coordinate of the point B. **(3)**

The finite region S enclosed by the curve and the x-axis is shown shaded in Figure 3. The region S is rotated through 2π radians about the x-axis.

(*b*) Find, by integration, the exact value of the volume of the solid generated. **(6)**

Ex4A p. 78-80

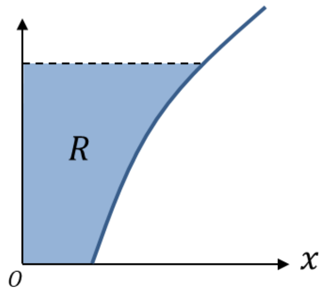
Revolving around the y-axis

Recap: When revolving around the y-axis, $\boldsymbol{V = \pi \int_b^a x^2 \, dy}$

i.e. we are just **swapping the roles of x and y**.

Example

The diagram shows the curve with equation $y = 4\ln x - 1$. The finite region R, shown in the diagram, is bounded by the curve, the x-axis, the y-axis and the line $y = 4$. Region R is rotated by 2π radians about the y-axis. Use integration to show that the exact value of the volume of the solid generated is $2\pi\sqrt{e}(e^2 - 1)$.



Ex4B p. 81-83

Volumes of revolution for parametric curves

We have seen in Pure Year 2 that parametric equations are where, instead of some single equation relating x and y, we have an equation for each of x and y in terms of some parameter, e.g. t. As t varies, this generates different points (x, y).

To integrate parametrically, the trick was to replace dx with $\frac{dx}{dt}\ dt$

$$V = \pi \int_{x=b}^{x=a} y^2\ dx \quad \Rightarrow$$

Note that as we're integrating with respect to t now, we need to find the equivalent limits for t. We can do the same for revolving around the y-axis: just replace dy with $\frac{dy}{dt}$ and change the limits.

Example

The curve C has parametric equations $x = t(1 + t)$, $y = \frac{1}{1+t}$, $t \geq 0$.

The region R is bounded by C, the x-axis and the lines $x = 0$ and $y = 0$. Find the exact volume of the solid formed when R is rotated 2π radians about the x-axis.

Test Your Understanding

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The finite shaded region S shown in Figure 3 is bounded by the curve C, the line $x = \sqrt{3}$ and the x-axis. This shaded region is rotated through 2π radians about the x-axis to form a solid of revolution.

(*c*) Find the volume of the solid of revolution, giving your answer in the form $p\pi\sqrt{3} + q\pi^2$, where p and q are constants.

(7)

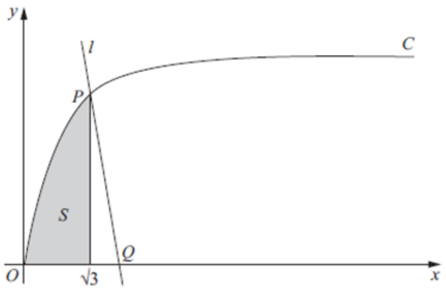


Figure 3

Figure 3 shows part of the curve C with parametric equations

$$x = \tan\theta, \qquad y = \sin\theta, \qquad 0 \leq \theta < \frac{\pi}{2}.$$

Ex4C p. 84-87

Modelling with Volumes of Revolution

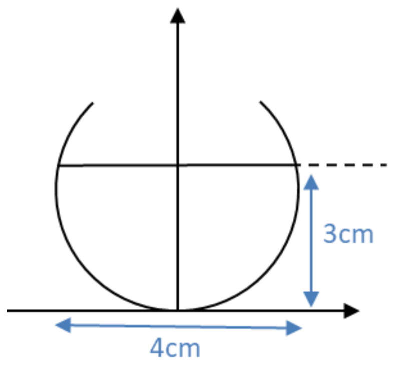
Example

The diagram shows a model of a goldfish bowl. The cross-section of the model is described by the curve with parametric equations
$x = 2\sin t\,,\ y = 2\cos t + 2, \frac{\pi}{6} \leq t \leq \frac{11\pi}{6}$, where the units of x and y are in cm. The goldfish bowl is formed by rotating this curve about the y-axis to form a solid of revolution.

(a) Find the volume of water required to fill the model to a height of 3cm.

The real goldfish bowl has a maximum diameter of 48cm.

(b) Find the volume of water required to fill the real goldfish bowl to the corresponding height.



Ex4D p. 88-89