Core Pure 2

Volumes of Revolution

Chapter Overview

1: Revolving around the -axis.

2: Revolving around the -axis.

3: Volumes of revolution with parametric curves.

4: Modelling



This chapter involves volumes of revolution but with trickier integration than in CP1.

Revolving around the x-axis

**Recap: When revolving around the -axis,**

Example

The region is bounded by the curve with equation , the -axis and . Find the volume of the solid formed when region is rotated through radians about the -axis.

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Ex4A p. 78-80

Revolving around the -axis

**Recap**: When revolving around the -axis,

i.e. we are just **swapping the roles of and** .

Example

The diagram shows the curve with equation . The finite region , shown in the diagram, is bounded by the curve, the -axis, the -axis and the line . Region is rotated by radians about the -axis. Use integration to show that the exact value of the volume of the solid generated is .



Ex4B p. 81-83

**Volumes of revolution for parametric curves**

We have seen in Pure Year 2 that parametric equations are where, instead of some single equation relating and , we have an equation for each of and in terms of some parameter, e.g. . As varies, this generates different points .

**To integrate parametrically, the trick was to replace with**

Note that as we’re integrating with respect to now, we need to find the equivalent limits for . We can do the same for revolving around the -axis: just replace with and change the limits.

Example

The curve has parametric equations , , .

The region is bounded by , the -axis and the lines and . Find the exact volume of the solid formed when is rotated radians about the -axis.

Test Your Understanding

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Ex4C p. 84-87

**Modelling with Volumes of Revolution**

Example

The diagram shows a model of a goldfish bowl. The cross-section of the model is described by the curve with parametric equations
, , where the units of and are in cm. The goldfish bowl is formed by rotating this curve about the -axis to form a solid of revolution.

1. Find the volume of water required to fill the model to a height of 3cm.

The real goldfish bowl has a maximum diameter of 48cm.

(b) Find the volume of water required to fill the real goldfish bowl to the corresponding height.

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Ex4D p. 88-89