CP1 Chapter 8

Proof by Induction

Chapter Overview

1. Summation Proofs
2. Divisibility Proofs
3. Matrix proofs



What is poof by induction?

We can often use **proof by induction** whenever we want to show some property holds for all integers (usually positive) up to infinity.

Example

Show that $\sum\_{i=1}^{n}i=\frac{1}{2}n\left(n+1\right)$ for all $n\in N$.

We can show this is true for certain examples. However, no number of examples are sufficient to show the statement is true for **all** integers $n$.

The key part of induction is that **we need to show that “if the statement is true for a particular value, then it is true for the next value**”.

Basic Example:

Prove by induction that if the first term of a sequence is $a\_{1}=3$ and $a\_{n+1}=a\_{n}+2$, then **every term in the sequence is odd**.

**Proof by induction:**

Step 1: **Basis:** Prove the general statement is true for $n=1$.

Step 2: **Assumption:** Assume the general statement is true for $n=k$.

Step 3: **Inductive:** Show that the general statement is then true for $n=k+1$.

Step 4: **Conclusion:** The general statement is then true for all positive integers $n$.

Type 1: Summation Proofs

Example 1: Show that $\sum\_{r=1}^{n}(2r-1)=n^{2}$ for all $n\in N$.

Basis step:

Assumption:

Inductive:

Conclusion:

Conclusion step – key points

(For method mark)

**Any 3 of these seen anywhere in the proof:**

* **“true for** $n=1$**”**
* **“assume true for** $n=k$**”**
* **“true for** $n=k+1$**”**
* **“true for all** $n$**/positive integers”**

Example 2. Prove by induction that for all positive integers $n$,

$$\sum\_{r=1}^{n}r2^{r}=2\left(1+\left(n-1\right)2^{n}\right)$$

Test Your Understanding



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Type 2: Divisibility Proofs

There are different techniques for the inductive step. For the following example we will consider 2 different methods.

Example

Prove by induction that $3^{2n}+11$ is divisible by 4 for all positive integers $n$.

Basis step:

Assumption:

Inductive step: method 1 Inductive step: method 2

Conclusion:

Example:

Prove by induction that $8^{n}-3^{n}$ is divisible by 5.

Question:

Prove by induction that $n^{3}-7n+9$ is divisible by 3 for all positive integers $n$.

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Type 3: Matrix Proofs

Example

Prove by induction that $\left(\begin{matrix}1&-1\\0&2\end{matrix}\right)^{n}=\left(\begin{matrix}1&1-2^{n}\\0&2^{n}\end{matrix}\right)$ for all $n\in Z^{+}$.

Example

Prove by induction that $\left(\begin{matrix}-2&9\\-1&4\end{matrix}\right)^{n}=\left(\begin{matrix}-3n+1&9n\\-n&3n+1\end{matrix}\right)$ for all $n\in Z^{+}$.

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Be the examiner!

How many marks does this deserve??



