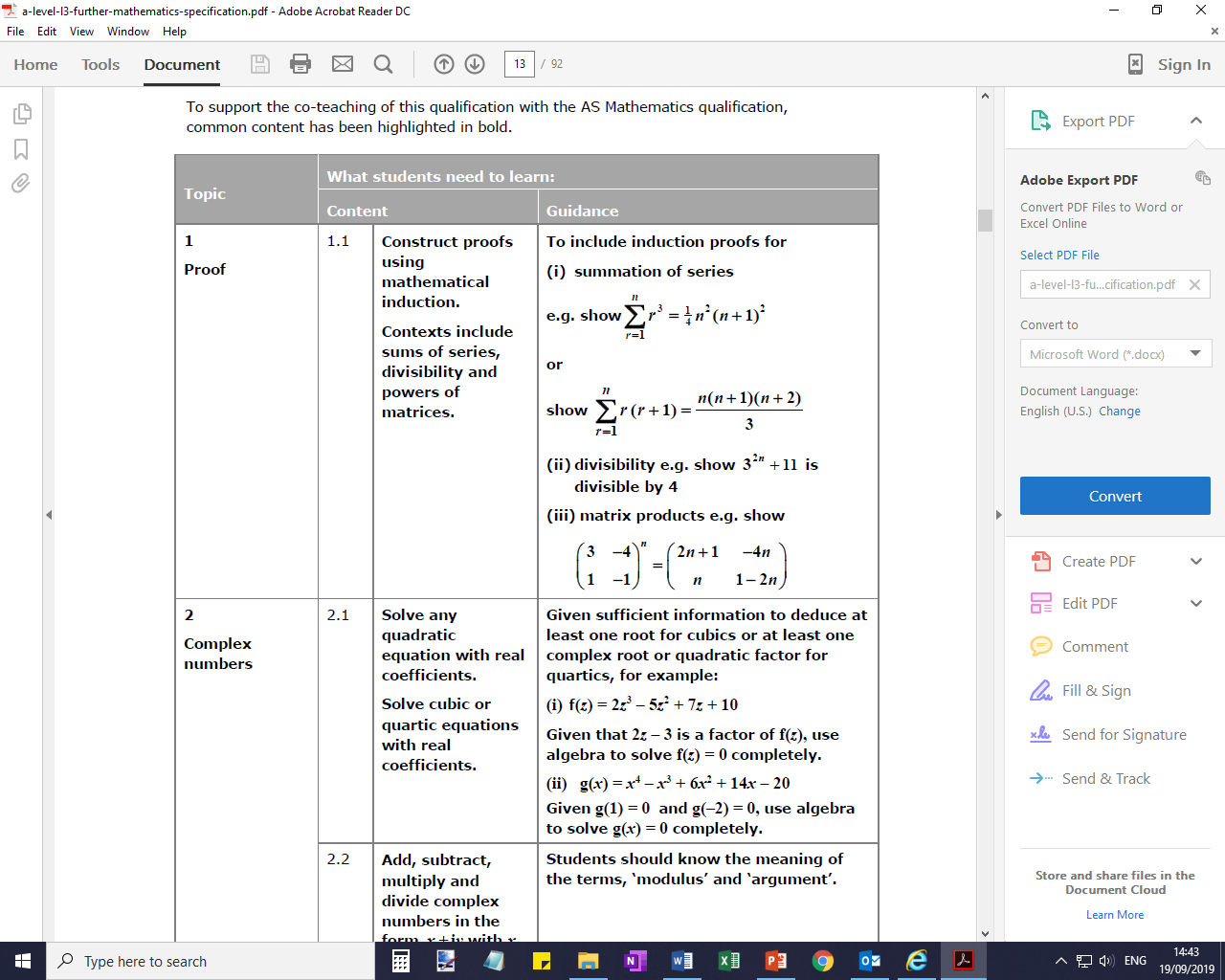
CP1 Chapter 8

Proof by Induction

Chapter Overview

1. Summation Proofs
2. Divisibility Proofs
3. Matrix proofs



What is poof by induction?

We can often use **proof by induction** whenever we want to show some property holds for all integers (usually positive) up to infinity.

Example

Show that for all .

We can show this is true for certain examples. However, no number of examples are sufficient to show the statement is true for **all** integers .

The key part of induction is that **we need to show that “if the statement is true for a particular value, then it is true for the next value**”.

Basic Example:

Prove by induction that if the first term of a sequence is and , then **every term in the sequence is odd**.

**Proof by induction:**

Step 1: **Basis:** Prove the general statement is true for .

Step 2: **Assumption:** Assume the general statement is true for .

Step 3: **Inductive:** Show that the general statement is then true for .

Step 4: **Conclusion:** The general statement is then true for all positive integers .

Type 1: Summation Proofs

Example 1: Show that for all .

Basis step:

Assumption:

Inductive:

Conclusion:

Conclusion step – key points

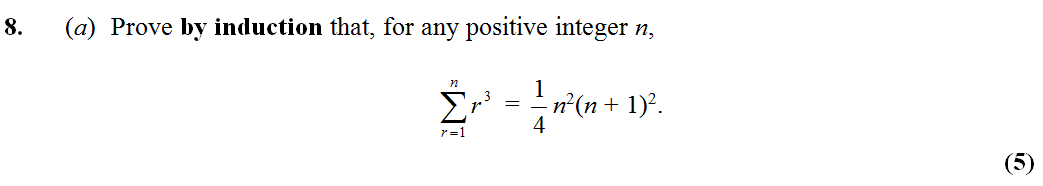
(For method mark)

**Any 3 of these seen anywhere in the proof:**

* **“true for ”**
* **“assume true for ”**
* **“true for ”**
* **“true for all /positive integers”**

Example 2. Prove by induction that for all positive integers ,

Test Your Understanding



Ex 8A Pages 158-159

Type 2: Divisibility Proofs

There are different techniques for the inductive step. For the following example we will consider 2 different methods.

Example

Prove by induction that is divisible by 4 for all positive integers .

Basis step:

Assumption:

Inductive step: method 1 Inductive step: method 2

Conclusion:

Example:

Prove by induction that is divisible by 5.

Question:

Prove by induction that is divisible by 3 for all positive integers .

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Type 3: Matrix Proofs

Example

Prove by induction that for all .

Example

Prove by induction that for all .

Ex 8C Pages 164

Be the examiner!

How many marks does this deserve??



