Core Pure 1

Complex Numbers

Chapter Overview

1: Understand and manipulate ($\times$, $\div$) complex numbers.

2: Find complex solutions to quadratic equations.

3: Find complex solutions to cubic and quartic equations.

2 **Complex numbers**	2.1	**Solve any quadratic equation with real coefficients.** **Solve cubic or quartic equations with real coefficients.**	**Given sufficient information to deduce at least one root for cubics or at least one complex root or quadratic factor for quartics, for example:** **(i)** $f(z) = 2z^3 - 5z^2 + 7z + 10$ **Given that** $2z - 3$ **is a factor of** $f(z)$**, use algebra to solve** $f(z) = 0$ **completely.** **(ii)** $g(x) = x^4 - x^3 + 6x^2 + 14x - 20$ **Given** $g(1) = 0$ **and** $g(-2) = 0$**, use algebra to solve** $g(x) = 0$ **completely.**
	2.2	**Add, subtract, multiply and divide complex numbers in the form** $x + iy$ **with** x **and** y **real.** **Understand and use the terms 'real part' and 'imaginary part'.**	**Students should know the meaning of the terms, 'modulus' and 'argument'.**

2 **Complex numbers** *continued*	2.3	**Understand and use the complex conjugate.** **Know that non-real roots of polynomial equations with real coefficients occur in conjugate pairs.**	**Knowledge that if** z_1 **is a root of** $f(z) = 0$ **then** z_1^* **is also a root.**
	2.4	**Use and interpret Argand diagrams.**	**Students should be able to represent the sum or difference of two complex numbers on an Argand diagram.**
	2.5	**Convert between the Cartesian form and the modulus-argument form of a complex number.**	Knowledge of radians is assumed.
	2.6	**Multiply and divide complex numbers in modulus argument form.**	**Knowledge of the results** $\lvert z_1 z_2 \rvert = \lvert z_1 \rvert \lvert z_2 \rvert$, $\left\lvert \frac{z_1}{z_2} \right\rvert = \frac{\lvert z_1 \rvert}{\lvert z_2 \rvert}$ $\arg(z_1 z_2) = \arg z_1 + \arg z_2$ $\arg\left(\frac{z_1}{z_2}\right) = \arg z_1 - \arg z_2$ Knowledge of radians and compound angle formulae is assumed.

Complex Number Basics

Examples: Write the following in terms of i

$\sqrt{(-36)} =$ $\sqrt{-\mathbf{1}} =$

$\sqrt{-4} =$ $\sqrt{-7} =$

Simplify:

$(2 + 3i) + (4 + i) =$

$i - 3(2 - i) =$

$\frac{10+4i}{2} =$

Solving Quadratic Equations

Examples

1. Solve $z^2 + 25 = 0$

2. Solve $z^2 + 3z + 5 = 0$

Exercise 1A/B
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Multiplying Complex Numbers

Examples

1. Express each of the following in the form $a + bi$, where a, b are integers.

a. $(2 + 3i)(3 - 2i)$

b. $(5 - 3i)^2$

2. Determine the value of i^3, i^4, i^{101} and $(3i)^5$

Test Your Understanding:

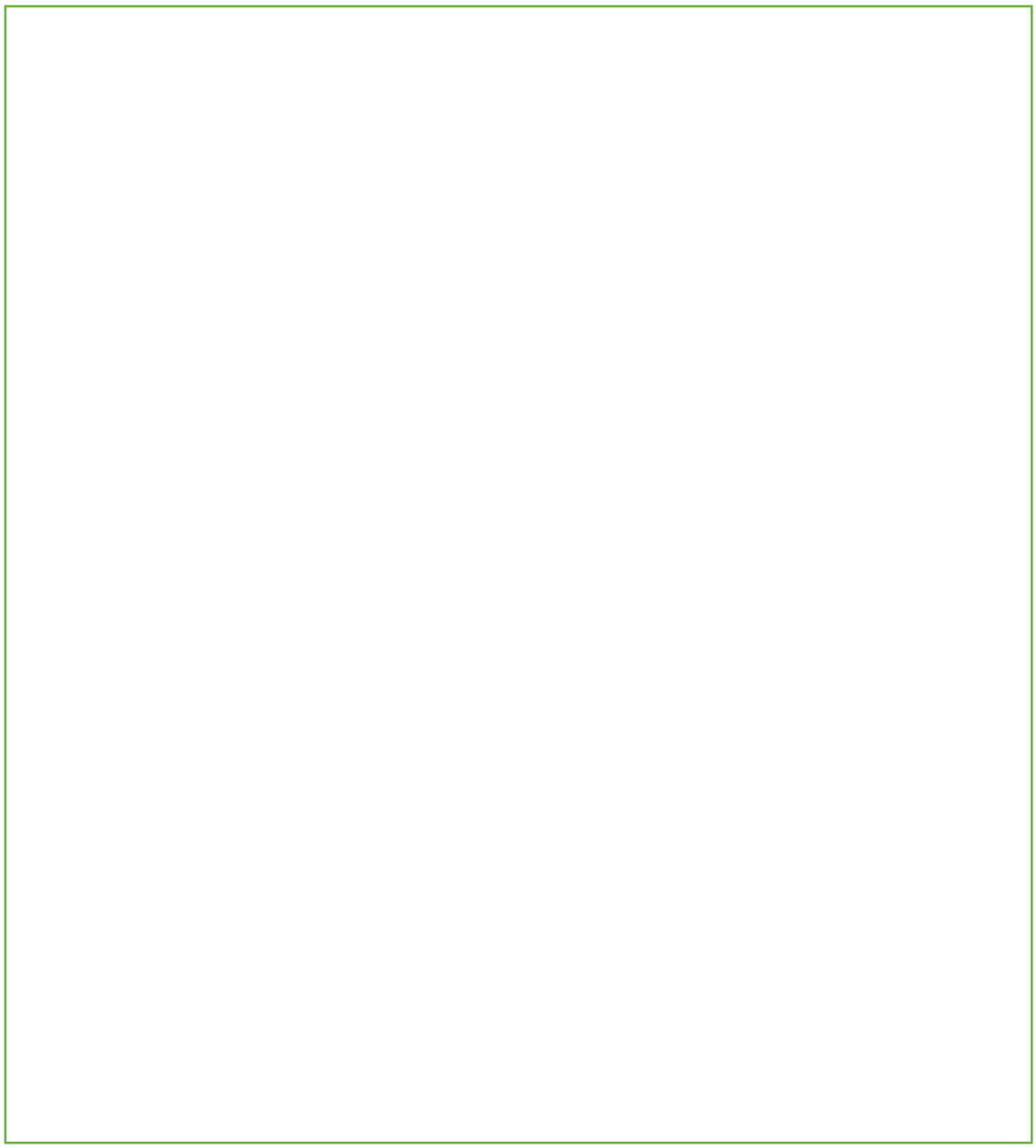
1. Edexcel FP1 June 2010

$$z = 2 - 3i$$

(*a*) Show that $z^2 = -5 - 12i$. **(2)**

2. Expand and simplify $(1 + i)^3$

Complex conjugates



Example:

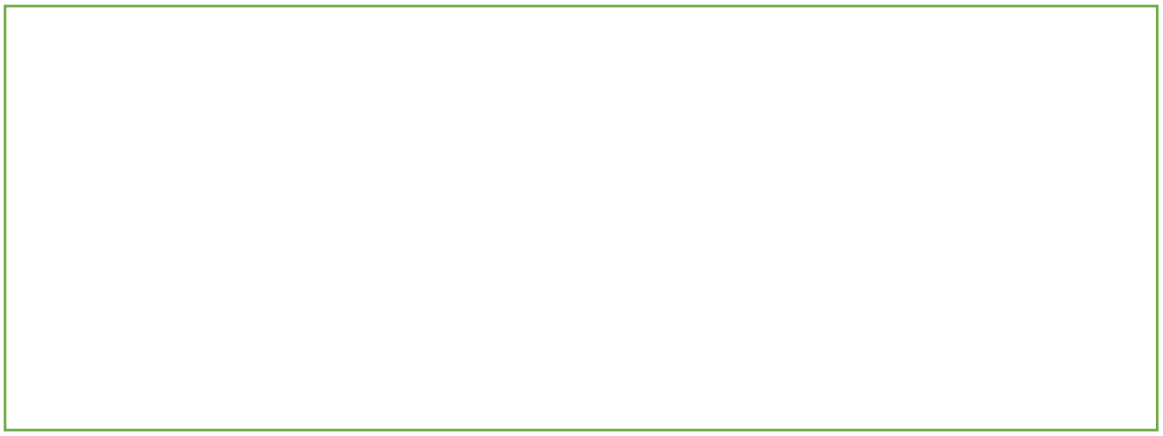
Write $\frac{5+4i}{2-3i}$ in the form $a + bi$

Test Your Understanding

Given that $z_1 = 3 + 2\mathrm{i}$ and $z_2 = \dfrac{12 - 5\mathrm{i}}{z_1}$,

(*a*) find z_2 in the form $a + \mathrm{i}b$, where a and b are real. **(2)**

Roots of Polynomials



Roots of Quadratics

Example:

Find the quadratic equation with roots α = 2+ 4i and β = 2 – 4i in the form $x^2 + ax + b = 0$

(2 Methods)

[Textbook] Given that $\alpha = 7 + 2i$ is one of the roots of a quadratic equation with real coefficients,

(a) state the value of the other root, β.

(b) find the quadratic equation.

Proof that Complex Roots Appear in Complex Pairs

Proof 1

Proof 2

Test Your Understanding

Given that $2 - 4i$ is a root of the equation

$$z^2 + pz + q = 0,$$

where p and q are real constants,

(a) write down the other root of the equation,

(1)

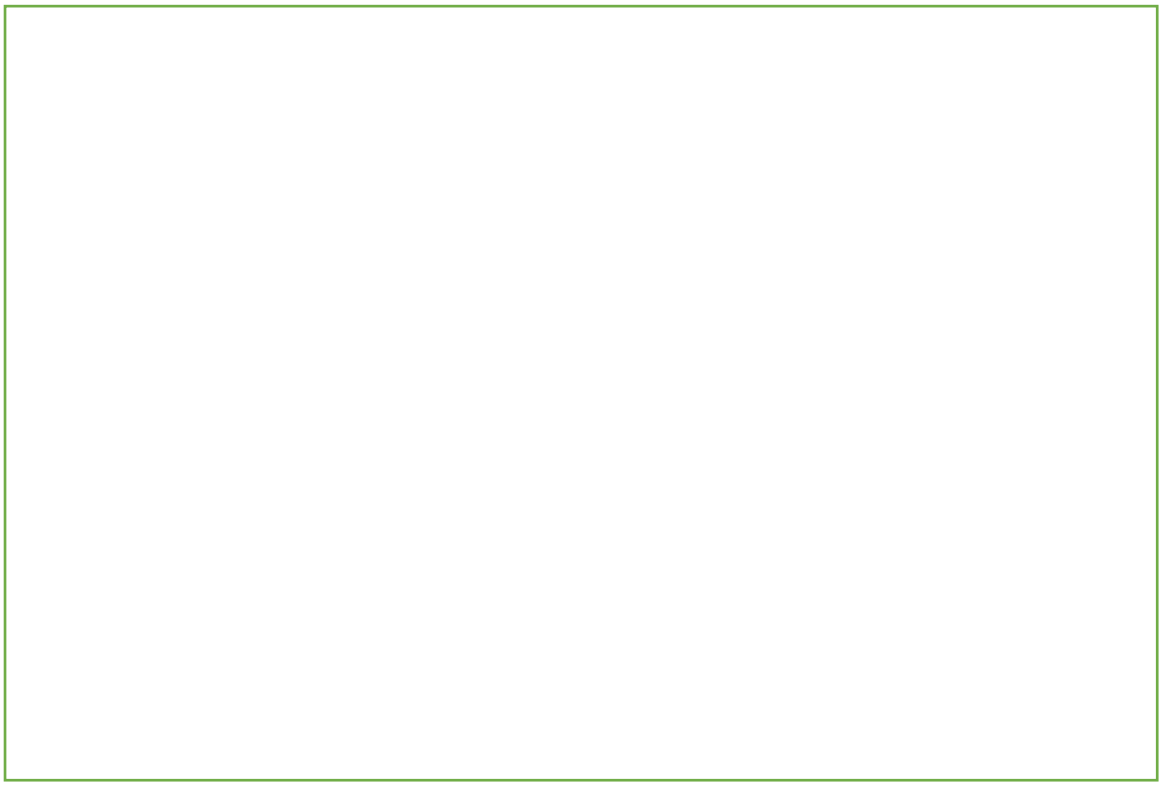
(b) find the value of p and the value of q.

(3)

Roots of Cubic and Quartic Equations

Cubics

Quartics



Examples

1. [Textbook] Given that $3 + i$ is a root of the quartic equation $2z^4 - 3z^3 - 39z^2 + 120z - 50 = 0$, solve the equation completely.

2. [Textbook] Show that $z^2 + 4$ is a factor of $z^4 - 2z^3 + 21z^2 - 8z + 68$. Hence solve the equation $z^4 - 2z^3 + 21z^2 - 8z + 68 = 0$

Test Your Understanding:

Given that 2 and 5 + 2i are roots of the equation

$$x^3 - 12x^2 + cx + d = 0, \quad c, d \in \mathbb{R},$$

(*a*) write down the other complex root of the equation.

(1)

(*b*) Find the value of c and the value of d.

(5)

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