9.6) Finding perpendiculars

Worked example	Your turn
Find the shortest distance between the parallel lines with equations: $\boldsymbol{r} = 2\boldsymbol{i} - \boldsymbol{j} + 6\boldsymbol{k} + \lambda(3\boldsymbol{i} + 4\boldsymbol{j} + 5\boldsymbol{k})$ and $\boldsymbol{r} = 3\boldsymbol{j} - \boldsymbol{k} + \mu(6\boldsymbol{i} + 8\boldsymbol{j} + 10\boldsymbol{k})$, Where λ and μ are scalars	Find the shortest distance between the parallel lines with equations: $\boldsymbol{r} = \boldsymbol{i} + 2\boldsymbol{j} - \boldsymbol{k} + \lambda(5\boldsymbol{i} + 4\boldsymbol{j} + 3\boldsymbol{k})$ and $\boldsymbol{r} = 2\boldsymbol{i} + \boldsymbol{k} + \mu(5\boldsymbol{i} + 4\boldsymbol{j} + 3\boldsymbol{k})$, Where λ and μ are scalars $\frac{21\sqrt{2}}{10}$

Worked example	Your turn
The lines l_1 and l_2 have equations $r = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$ and $r = \begin{pmatrix} 1 \\ 3 \\ -1 \end{pmatrix} + \mu \begin{pmatrix} -1 \\ -1 \\ 2 \end{pmatrix}$ respectively, where λ and μ are scalars. Find the shortest distance between these two lines.	The lines l_1 and l_2 have equations $r = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$ and $r = \begin{pmatrix} -1 \\ 3 \\ -1 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ -1 \\ -1 \end{pmatrix}$ respectively, where λ and μ are scalars. Find the shortest distance between these two lines. $2\sqrt{2}$

Worked example

The line l has equation $\frac{x-1}{-2} = \frac{y-1}{2} = \frac{z+3}{1}$, and the point A has coordinates $(-1,2,1)$.

(a) Find the shortest distance between A and l.

(b) Find the Cartesian equation of the line that is perpendicular to l and passes through A.

Your turn

The line l has equation $\frac{x-1}{2} = \frac{y-1}{-2} = \frac{z+3}{-1}$, and the point A has coordinates $(1,2,-1)$.

(a) Find the shortest distance between A and l.

(b) Find the Cartesian equation of the line that is perpendicular to l and passes through A.

(a) $\frac{\sqrt{29}}{3}$

(b) $\frac{x-1}{8} = \frac{y-2}{1} = \frac{z+1}{14}$