9.5) Points of intersection

Worked example

The lines l_1 and l_2 have vector equations $r = i + 2j + 3k + \lambda(4i - 3j - 2k)$ and $r = -3i - 2k + \mu(-4i + 8j + 9k)$ respectively. Show that the two lines intersect, and find the position vector of the point of intersection.

Your turn

The lines l_1 and l_2 have vector equations $r = 3i + j + k + \lambda(i - 2j - k)$ and $r = -2j + 3k + \mu(-5i + j + 4k)$ respectively. Show that the two lines intersect, and find the position vector of the point of intersection.

$(5, -3, -1)$

Worked example

Find the point of intersection of the line l and the plane Π where:

l: $\boldsymbol{r} = -2\boldsymbol{i} + \boldsymbol{j} + 3\boldsymbol{k} + \lambda(2\boldsymbol{i} - \boldsymbol{j} + 7\boldsymbol{k})$

Π: $\boldsymbol{r} \cdot (3\boldsymbol{i} - 2\boldsymbol{j} + \boldsymbol{k}) = 5$

Your turn

Find the point of intersection of the line l and the plane Π where:

l: $\boldsymbol{r} = -\boldsymbol{i} + \boldsymbol{j} - 5\boldsymbol{k} + \lambda(\boldsymbol{i} + \boldsymbol{j} + 2\boldsymbol{k})$

Π: $\boldsymbol{r} \cdot (\boldsymbol{i} + 2\boldsymbol{j} + 3\boldsymbol{k}) = 4$

$(1, 3, -1)$

Worked example

A line has Cartesian equation

$$\frac{x-3}{-5}=\frac{y+2}{-3}=\frac{4-z}{-1}$$

A plane Π has Cartesian equation

$$-4x-3y+2z=10$$

Find the position vector of the point of intersection of the line and the plane.

Your turn

A line has Cartesian equation

$$\frac{x-3}{5}=\frac{y+2}{3}=\frac{4-z}{1}$$

A plane Π has Cartesian equation

$$4x+3y-2z=-10$$

Find the position vector of the point of intersection of the line and the plane.

$$\left(\frac{53}{31},-\frac{86}{31},\frac{132}{32}\right)$$

Worked example

The lines l_1 and l_2 have equations $\frac{x-3}{2} = y - 4 = \frac{z-1}{-5}$ and $\frac{x-1}{3} = \frac{y+2}{-7} = z - 5$ respectively. Prove that l_1 and l_2 are skew.

Your turn

The lines l_1 and l_2 have equations $\frac{x-2}{4} = \frac{y+3}{2} = z - 1$ and $\frac{x+1}{5} = \frac{y}{4} = \frac{z-4}{-2}$ respectively. Prove that l_1 and l_2 are skew.

Proof