9.4) Calculating angles between lines and planes

Worked example

Find the acute angle between the line l with equation $\boldsymbol{r} = -5\boldsymbol{i} + \boldsymbol{j} + 2\boldsymbol{k} + \lambda(-12\boldsymbol{i} + 4\boldsymbol{j} + 3\boldsymbol{k})$ and the plane with equation $\boldsymbol{r} \cdot (-\boldsymbol{i} - 2\boldsymbol{j} + 2\boldsymbol{k}) = 2.$

Your turn

Find the acute angle between the line l with equation $\boldsymbol{r} = 2\boldsymbol{i} + \boldsymbol{j} - 5\boldsymbol{k} + \lambda(3\boldsymbol{i} + 4\boldsymbol{j} - 12\boldsymbol{k})$ and the plane with equation $\boldsymbol{r} \cdot (2\boldsymbol{i} - 2\boldsymbol{j} - \boldsymbol{k}) = 2.$

$14.9°$ (3 sf)

Worked example

The plane P has equation

$$r = \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 2 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 2 \\ 3 \end{pmatrix}$$

a) Find a vector perpendicular to the plane P

b) The line l passes through the point A (3, 3, 1) and meets P at (2, 1, 3). The acute angle between the plane P and the line l is θ. Find θ to the nearest degree

Your turn

The plane P has equation

$$r = \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 2 \\ -1 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ 2 \\ 2 \end{pmatrix}$$

a) Find a vector perpendicular to the plane P

b) The line l passes through the point A (1, 3, 3) and meets P at (3, 1, 2). The acute angle between the plane P and the line l is θ. Find θ to the nearest degree

$\theta = 63°$ (nearest degree)

Worked example

Find the acute angle between the planes:

$\Pi_1: \ \boldsymbol{r} \cdot (3\boldsymbol{i} - \boldsymbol{j} + \boldsymbol{k}) = 4$

$\Pi_2: \ \boldsymbol{r} \cdot (2\boldsymbol{i} + 3\boldsymbol{j}) = 7$

Your turn

Find the acute angle between the planes:

$\Pi_1: \ \boldsymbol{r} \cdot (4\boldsymbol{i} + 4\boldsymbol{j} - 7\boldsymbol{k}) = 13$

$\Pi_2: \ \boldsymbol{r} \cdot (7\boldsymbol{i} - 4\boldsymbol{j} + 4\boldsymbol{k}) = 6$

$\theta = 78.6°$ (1 dp)

Worked example

The lines l_1 and l_2 have Cartesian equations $\frac{x+6}{-1} = \frac{y-3}{2} = \frac{z-2}{3}$ and $\frac{x+8}{-2} = \frac{y-4}{3} = \frac{z+13}{-1}$ respectively.

(a) Show that the point $A\ (-2, -5, -10)$ lies on both lines

(b) Find the size of the acute angle between the lines at A

Your turn

The lines l_1 and l_2 have Cartesian equations $\frac{x-6}{-1} = \frac{y+3}{2} = \frac{z+2}{3}$ and $\frac{x+5}{2} = \frac{y-15}{-3} = \frac{z-3}{1}$ respectively.

(a) Show that the point $A\ (3, 3, 7)$ lies on both lines

(b) Find the size of the acute angle between the lines at A

(a) Shown

(b) 69.1° (1 dp)