9.3) Scalar product

Worked example	Your turn
$\begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 4 \\ 0 \\ 1 \end{pmatrix}$	$\begin{pmatrix} 5 \\ -2 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ 3 \\ 1 \end{pmatrix}$ 9
$\begin{pmatrix} a \\ 5 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ b \\ 10 \end{pmatrix}$	

Worked example	Your turn
Find the acute angle between the vectors $\boldsymbol{a} = \begin{pmatrix} 5 \\ 3 \\ 1 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} 5 \\ 0 \\ 1 \end{pmatrix}$.	Find the acute angle between the vectors $\boldsymbol{a} = \begin{pmatrix} 5 \\ 3 \\ 1 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} 1 \\ 0 \\ 5 \end{pmatrix}$. 70.64° (2 dp)
Find the angle between the vectors $2\boldsymbol{i} + 4\boldsymbol{j} - \boldsymbol{k}$ and $\boldsymbol{j} + 8\boldsymbol{k}$ $\begin{pmatrix} 2 \\ 4 \\ -1 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 1 \\ 8 \end{pmatrix}$	Find the angle between the vectors $-\boldsymbol{i} + \boldsymbol{j} + 3\boldsymbol{k}$ and $7\boldsymbol{i} - 2\boldsymbol{j} + 2\boldsymbol{k}$ 96.9° (3 sf)

Worked example	Your turn
If $A(5,3,2)$, $B(4,0,5)$ and $C(2,-3,4)$, find the area of triangle ABC	If $A(2,3,5)$, $B(5,0,4)$ and $C(4,-3,2)$, find the area of triangle ABC 7.10 (3 sf)

Worked example	Your turn
Show that $\boldsymbol{a} = \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} -1 \\ 7 \\ -2 \end{pmatrix}$ are perpendicular.	Show that $\boldsymbol{a} = \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} 1 \\ 0 \\ -2 \end{pmatrix}$ are perpendicular. Shown

Worked example	Your turn
Given that the vectors $a = 1\boldsymbol{i} - 6\boldsymbol{j} + \boldsymbol{2k}$ and $b = \lambda\boldsymbol{i} - 2\boldsymbol{j} + 5\boldsymbol{k}$ are perpendicular, find the value of λ	Given that the vectors $a = 2\boldsymbol{i} - 6\boldsymbol{j} + \boldsymbol{k}$ and $b = 5\boldsymbol{i} + 2\boldsymbol{j} + \lambda\boldsymbol{k}$ are perpendicular, find the value of λ $\lambda = 2$

Worked example	Your turn
Show that the two lines are perpendicular: l_1: $r = (-9\boldsymbol{i} + 10\boldsymbol{k}) + \lambda(3\boldsymbol{i} + \boldsymbol{2j} - \boldsymbol{k})$ l_2: $r = (-3\boldsymbol{i} + 5\boldsymbol{k}) + \mu(2\boldsymbol{i} - \boldsymbol{j} + 4\boldsymbol{k})$	Show that the two lines are perpendicular: l_1: $r = (-9\boldsymbol{i} + 10\boldsymbol{k}) + \lambda(2\boldsymbol{i} + \boldsymbol{j} - \boldsymbol{k})$ l_2: $r = (-3\boldsymbol{i} + 5\boldsymbol{k}) + \mu(3\boldsymbol{i} - \boldsymbol{j} + 5\boldsymbol{k})$ Shown

Worked example	Your turn
Given that $\boldsymbol{a} = 2\boldsymbol{i} - 5\boldsymbol{j} + 4\boldsymbol{k}$ and $\boldsymbol{b} = -4 + 8\boldsymbol{j} - 5\boldsymbol{k}$, find a vector which is perpendicular to both $\boldsymbol{a}$ and $\boldsymbol{b}$.	Given that $\boldsymbol{a} = -2\boldsymbol{i} + 5\boldsymbol{j} - 4\boldsymbol{k}$ and $\boldsymbol{b} = 4\boldsymbol{i} - 8\boldsymbol{j} + 5\boldsymbol{k}$, find a vector which is perpendicular to both $\boldsymbol{a}$ and $\boldsymbol{b}$. $7\boldsymbol{i} + 6\boldsymbol{j} + 4\boldsymbol{k}$

Worked example	Your turn
Find, to the nearest tenth of a degree, the angle that the vector $9\boldsymbol{i} - 5\boldsymbol{j} + 3\boldsymbol{k}$ makes with: • The positive x-axis • The positive y-axis	Find, to the nearest tenth of a degree, the angle that the vector $\boldsymbol{i} + 11\boldsymbol{j} - 4\boldsymbol{k}$ makes with: • The positive z-axis $109.9°$ (1 dp)

Worked example	Your turn
The points P, Q and R have coordinates $(6, -1, 1), (4, 5, -2)$ and $(-5, 3, 0)$ respectively. (a) Show that PQ is perpendicular to QR (b) Hence find the centre and radius of the circle that passes through points P, Q and R	The points P, Q and R have coordinates $(1, -1, 6), (-2, 5, 4)$ and $(0, 3, -5)$ respectively. (a) Show that PQ is perpendicular to QR (b) Hence find the centre and radius of the circle that passes through points P, Q and R (a) Shown (b) Centre $\left(\frac{1}{2}, 1, \frac{1}{2}\right)$, radius $\frac{\sqrt{138}}{2}$