9.1) Equation of a line in three dimensions

Worked example

The straight line has vector equation $r = (i + 5j - 3k) + t(2i - j - 6k)$. Given that the point $(0, a, b)$ lines on l, find the value of a and the value of b.

Your turn

The straight line has vector equation $r = (3i + 2j - 5k) + t(i - 6j - 2k)$. Given that the point $(a, b, 0)$ lines on l, find the value of a and the value of b.

$$a = \frac{1}{2}, b = 17$$

Worked example

The straight line l has vector equation $\boldsymbol{r} = (1i + 4j - 3k) + \lambda(10i - 15j + 5k)$.
Show that another vector equation of l is

$$\boldsymbol{r} = (7i - 5j) + \mu(2i - 3j + k)$$

Your turn

The straight line l has vector equation $\boldsymbol{r} = (2i + 5j - 3k) + \lambda(6i - 2j + 4k)$.
Show that another vector equation of l is

$$\boldsymbol{r} = (8i + 3j + k) + \mu(3i - j + 2k)$$

$$\begin{pmatrix} 6 \\ -2 \\ 4 \end{pmatrix} = 2\begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix}$$

∴ lines parallel

Let $\lambda = 0$ -> r $= \begin{pmatrix} 2 \\ 5 \\ -3 \end{pmatrix}$

Let $\mu = -2$ -> $r = \begin{pmatrix} 8 + 3(-2) \\ 3 - (-2) \\ 1 + 2(-2) \end{pmatrix} = \begin{pmatrix} 2 \\ 5 \\ -3 \end{pmatrix}$

∴ point in common

Lines parallel and shared point ∴ same line

Worked example

The line l has equation $r = \begin{pmatrix} 4 \\ 1 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 2 \\ -1 \end{pmatrix}$, and the point P has position vector $\begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix}$.

(a) Show that P does not lie on l.

Given that a circle, centre P, intersects l at points A and B, and that A has position vector $\begin{pmatrix} -6 \\ 3 \\ 0 \end{pmatrix}$,

(b) find the possible position vectors of B.

Your turn

The line l has equation $r = \begin{pmatrix} -2 \\ 1 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$, and the point P has position vector $\begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$.

(a) Show that P does not lie on l.

Given that a circle, centre P, intersects l at points A and B, and that A has position vector $\begin{pmatrix} 0 \\ -3 \\ 6 \end{pmatrix}$,

(b) find the possible position vectors of B.

(a) Shown

(b) $(-3, 3, 3)$ or $(0, -3, 6)$