9) Vectors

9.1) Equation of a line in three dimensions

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Worked example	Your turn
The straight line has vector equation $r = (i + 5j - 3k) + t(2i - j - 6k)$. Given that the point $(0, a, b)$ lines on l, find the value of a and the value of b.	The straight line has vector equation $r = (3i + 2j - 5k) + t(i - 6j - 2k)$. Given that the point $(a, b, 0)$ lines on l, find the value of a and the value of b. $a = \frac{1}{2}, b = 17$

Worked example	Your turn
The straight line l has vector equation $\boldsymbol{r} = (1i + 4j - 3k) + \lambda(10i - 15j + 5k)$. Show that another vector equation of l is $\boldsymbol{r} = (7i - 5j) + \mu(2i - 3j + k)$	The straight line l has vector equation $\boldsymbol{r} = (2i + 5j - 3k) + \lambda(6i - 2j + 4k)$. Show that another vector equation of l is $\boldsymbol{r} = (8i + 3j + k) + \mu(3i - j + 2k)$ $\begin{pmatrix} 6 \\ -2 \\ 4 \end{pmatrix} = 2\begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix}$ ∴ lines parallel Let $\lambda = 0$ -> $\mathrm{r} = \begin{pmatrix} 2 \\ 5 \\ -3 \end{pmatrix}$ Let $\mu = -2$ -> $r = \begin{pmatrix} 8 + 3(-2) \\ 3 - (-2) \\ 1 + 2(-2) \end{pmatrix} = \begin{pmatrix} 2 \\ 5 \\ -3 \end{pmatrix}$ ∴ point in common Lines parallel and shared point ∴ same line

Worked example	Your turn
The line l has equation $r = \begin{pmatrix} 4 \\ 1 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 2 \\ -1 \end{pmatrix}$, and the point P has position vector $\begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix}$. (a) Show that P does not lie on l. Given that a circle, centre P, intersects l at points A and B, and that A has position vector $\begin{pmatrix} -6 \\ 3 \\ 0 \end{pmatrix}$, (b) find the possible position vectors of B.	The line l has equation $r = \begin{pmatrix} -2 \\ 1 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$, and the point P has position vector $\begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$. (a) Show that P does not lie on l. Given that a circle, centre P, intersects l at points A and B, and that A has position vector $\begin{pmatrix} 0 \\ -3 \\ 6 \end{pmatrix}$, (b) find the possible position vectors of B. (a) Shown (b) $(-3, 3, 3)$ or $(0, -3, 6)$

Worked example	Your turn
Find the Cartesian equation of the line with vector equation $\boldsymbol{r} = \begin{pmatrix} -4 \\ 2 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -2 \\ -5 \end{pmatrix}$.	Find the Cartesian equation of the line with vector equation $\boldsymbol{r} = \begin{pmatrix} 4 \\ 3 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 2 \\ 5 \end{pmatrix}$. $$\frac{x-4}{-1} = \frac{y-3}{2} = \frac{z+2}{5}$$
Find the Cartesian equation of the line with vector equation $\boldsymbol{r} = \begin{pmatrix} -2 \\ 0 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix}$.	Find the Cartesian equation of the line with vector equation $\boldsymbol{r} = \begin{pmatrix} 2 \\ 5 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 3 \\ -2 \end{pmatrix}$. $$\frac{x-2}{1} = \frac{y-5}{3} = \frac{z}{-2}$$

Worked example

The Cartesian equation of a line is $\frac{x+2}{-3} = \frac{y}{-4} = \frac{z-5}{1}$.
Find the vector form of the equation of the line.

The Cartesian equation of a line is $\frac{2-x}{3} = \frac{y+3}{-1} = \frac{5-z}{-4}$.
Find the vector form of the equation of the line.

Your turn

The Cartesian equation of a line is $\frac{x-2}{3} = \frac{y+5}{-1} = \frac{z}{4}$.
Find the vector form of the equation of the line.

$$r = \begin{pmatrix} 2 \\ -5 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ -1 \\ 4 \end{pmatrix}$$

Worked example	Your turn
Show that the points $A\ (3, -4, -5), B(-3, 1, -2)$ and $C\ (-9, 6, 1)$ are collinear	Show that the points $A\ (-3, 4, 5), B(3, -1, 2)$ and $C\ (9, 2, -1)$ are collinear Shown

Worked example	Your turn
The Cartesian equation of a line is $y = 2x - 3$. Find the vector form of the equation of the line.	The Cartesian equation of a line is $y = 3x + 2$. Find the vector form of the equation of the line. $\begin{pmatrix} 0 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 3 \end{pmatrix}$
The Cartesian equation of a line is $y = 3x - 2$. Find the vector form of the equation of the line.	

9.2) Equation of a plane in three dimensions

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Worked example	Your turn
A plane Π passes through the points $A(1,-2,2), B(-3,-2,1), C(5,4,3)$ Find the equation of the plane Π in the form $\mathrm{r} = \boldsymbol{a} + \lambda\boldsymbol{b} + \mu\boldsymbol{c}$	A plane Π passes through the points $A(2,2-1), B(3,2,-1), C(4,3,5)$ Find the equation of the plane Π in the form $\mathrm{r} = \boldsymbol{a} + \lambda\boldsymbol{b} + \mu\boldsymbol{c}$ $r = 2\boldsymbol{i} + 2\boldsymbol{j} - \boldsymbol{k} + \lambda\boldsymbol{i} + \mu(2\boldsymbol{i} + \boldsymbol{j} + 6\boldsymbol{k})$ $r = \begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix} + \lambda\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + \mu\begin{pmatrix} 2 \\ 1 \\ 6 \end{pmatrix}$

Worked example

Verify that the point P with position vector $\begin{pmatrix} 2 \\ -3 \\ 2 \end{pmatrix}$ lies in the plane with vector equation

$$r = \begin{pmatrix} 3 \\ 4 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$$

Your turn

Verify that the point P with position vector $\begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix}$ lies in the plane with vector equation

$$r = \begin{pmatrix} 3 \\ 4 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$$

Shown

Worked example	Your turn
The plane Π is perpendicular to the normal $\boldsymbol{n} = 2\boldsymbol{i} - \boldsymbol{j} + \boldsymbol{3k}$ and passes through the point P with position vector $4\boldsymbol{i} - \boldsymbol{8j} + 7\boldsymbol{k}$. Find the Cartesian equation of Π.	The plane Π is perpendicular to the normal $\boldsymbol{n} = 3\boldsymbol{i} - 2\boldsymbol{j} + \boldsymbol{k}$ and passes through the point P with position vector $8\boldsymbol{i} + 4\boldsymbol{j} - 7\boldsymbol{k}$. Find the Cartesian equation of Π. $3x - 2y + z = 9$

Worked example	Your turn
Show that the points $(3, 2, 2)$, $(3, 5, 1)$, $(-1, 3, 4)$ and $(-1, 6, 3)$ are coplanar.	Show that the points $(2, 2, 3)$, $(1, 5, 3)$, $(4, 3, -1)$ and $(3, 6, -1)$ are coplanar. Shown

Worked example	Your turn
Show that the points $(3, 2, 2)$, $(3, 5, 1)$, $(-1, 3, 4)$ and $(-1, 6, 4)$ are not coplanar.	Show that the points $(2, 2, 3)$, $(1, 5, 3)$, $(4, 3, -1)$ and $(4, 6, -1)$ are coplanar. Shown

9.3) Scalar product

Worked example	Your turn
$\begin{pmatrix}3\\2\\1\end{pmatrix}\cdot\begin{pmatrix}4\\0\\1\end{pmatrix}$	$\begin{pmatrix}5\\-2\\0\end{pmatrix}\cdot\begin{pmatrix}3\\3\\1\end{pmatrix}$ 9
$\begin{pmatrix}a\\5\\1\end{pmatrix}\cdot\begin{pmatrix}3\\b\\10\end{pmatrix}$	

Worked example	Your turn
Find the acute angle between the vectors $\boldsymbol{a} = \begin{pmatrix} 5 \\ 3 \\ 1 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} 5 \\ 0 \\ 1 \end{pmatrix}$.	Find the acute angle between the vectors $\boldsymbol{a} = \begin{pmatrix} 5 \\ 3 \\ 1 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} 1 \\ 0 \\ 5 \end{pmatrix}$. 70.64° (2 dp)
Find the angle between the vectors $2\boldsymbol{i} + 4\boldsymbol{j} - \boldsymbol{k}$ and $\boldsymbol{j} + 8\boldsymbol{k}$ $\begin{pmatrix} 2 \\ 4 \\ -1 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 1 \\ 8 \end{pmatrix}$	Find the angle between the vectors $-\boldsymbol{i} + \boldsymbol{j} + 3\boldsymbol{k}$ and $7\boldsymbol{i} - 2\boldsymbol{j} + 2\boldsymbol{k}$ 96.9° (3 sf)

Worked example	Your turn
If $A(5,3,2)$, $B(4,0,5)$ and $C(2,-3,4)$, find the area of triangle ABC	If $A(2,3,5)$, $B(5,0,4)$ and $C(4,-3,2)$, find the area of triangle ABC 7.10 (3 sf)

Worked example	Your turn
Show that $\boldsymbol{a} = \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} -1 \\ 7 \\ -2 \end{pmatrix}$ are perpendicular.	Show that $\boldsymbol{a} = \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} 1 \\ 0 \\ -2 \end{pmatrix}$ are perpendicular. Shown

Worked example	Your turn
Given that the vectors $a = 1\boldsymbol{i} - 6\boldsymbol{j} + \boldsymbol{2k}$ and $b = \lambda\boldsymbol{i} - 2\boldsymbol{j} + 5\boldsymbol{k}$ are perpendicular, find the value of λ	Given that the vectors $a = 2\boldsymbol{i} - 6\boldsymbol{j} + \boldsymbol{k}$ and $b = 5\boldsymbol{i} + 2\boldsymbol{j} + \lambda\boldsymbol{k}$ are perpendicular, find the value of λ $\lambda = 2$

Worked example	Your turn
Show that the two lines are perpendicular: l_1: $r = (-9\boldsymbol{i} + 10\boldsymbol{k}) + \lambda(3\boldsymbol{i} + \mathbf{2}\boldsymbol{j} - \boldsymbol{k})$ l_2: $r = (-3\boldsymbol{i} + 5\boldsymbol{k}) + \mu(2\boldsymbol{i} - \boldsymbol{j} + 4\boldsymbol{k})$	Show that the two lines are perpendicular: l_1: $r = (-9\boldsymbol{i} + 10\boldsymbol{k}) + \lambda(2\boldsymbol{i} + \boldsymbol{j} - \boldsymbol{k})$ l_2: $r = (-3\boldsymbol{i} + 5\boldsymbol{k}) + \mu(3\boldsymbol{i} - \boldsymbol{j} + 5\boldsymbol{k})$ Shown

Worked example	Your turn
Given that $\boldsymbol{a} = 2\boldsymbol{i} - 5\boldsymbol{j} + 4\boldsymbol{k}$ and $\boldsymbol{b} = -4 + 8\boldsymbol{j} - 5\boldsymbol{k}$, find a vector which is perpendicular to both $\boldsymbol{a}$ and $\boldsymbol{b}$.	Given that $\boldsymbol{a} = -2\boldsymbol{i} + 5\boldsymbol{j} - 4\boldsymbol{k}$ and $\boldsymbol{b} = 4\boldsymbol{i} - 8\boldsymbol{j} + 5\boldsymbol{k}$, find a vector which is perpendicular to both $\boldsymbol{a}$ and $\boldsymbol{b}$. $7\boldsymbol{i} + 6\boldsymbol{j} + 4\boldsymbol{k}$

Worked example	Your turn
Find, to the nearest tenth of a degree, the angle that the vector $9\boldsymbol{i} - 5\boldsymbol{j} + 3\boldsymbol{k}$ makes with: • The positive x-axis • The positive y-axis	Find, to the nearest tenth of a degree, the angle that the vector $\boldsymbol{i} + 11\boldsymbol{j} - 4\boldsymbol{k}$ makes with: • The positive z-axis $109.9°$ (1 dp)

Worked example	Your turn
The points P, Q and R have coordinates $(6, -1, 1), (4, 5, -2)$ and $(-5, 3, 0)$ respectively. (a) Show that PQ is perpendicular to QR (b) Hence find the centre and radius of the circle that passes through points P, Q and R	The points P, Q and R have coordinates $(1, -1, 6), (-2, 5, 4)$ and $(0, 3, -5)$ respectively. (a) Show that PQ is perpendicular to QR (b) Hence find the centre and radius of the circle that passes through points P, Q and R (a) Shown (b) Centre $\left(\frac{1}{2}, 1, \frac{1}{2}\right)$, radius $\frac{\sqrt{138}}{2}$

9.4) Calculating angles between lines and planes

Worked example	Your turn
Find the acute angle between the line l with equation $\boldsymbol{r} = -5\boldsymbol{i} + \boldsymbol{j} + 2\boldsymbol{k} + \lambda(-12\boldsymbol{i} + 4\boldsymbol{j} + 3\boldsymbol{k})$ and the plane with equation $\boldsymbol{r} \cdot (-\boldsymbol{i} - \mathbf{2}\boldsymbol{j} + 2\boldsymbol{k}) = 2$.	Find the acute angle between the line l with equation $\boldsymbol{r} = 2\boldsymbol{i} + \boldsymbol{j} - 5\boldsymbol{k} + \lambda(3\boldsymbol{i} + 4\boldsymbol{j} - 12\boldsymbol{k})$ and the plane with equation $\boldsymbol{r} \cdot (2\boldsymbol{i} - 2\boldsymbol{j} - \boldsymbol{k}) = 2$. 14.9° (3 sf)

Worked example	Your turn
The plane P has equation $$r = \begin{pmatrix}2\\1\\3\end{pmatrix} + \lambda\begin{pmatrix}-1\\2\\0\end{pmatrix} + \mu\begin{pmatrix}2\\2\\3\end{pmatrix}$$ a) Find a vector perpendicular to the plane P b) The line l passes through the point A (3, 3, 1) and meets P at (2, 1, 3). The acute angle between the plane P and the line l is θ. Find θ to the nearest degree	The plane P has equation $$r = \begin{pmatrix}3\\1\\2\end{pmatrix} + \lambda\begin{pmatrix}0\\2\\-1\end{pmatrix} + \mu\begin{pmatrix}3\\2\\2\end{pmatrix}$$ a) Find a vector perpendicular to the plane P b) The line l passes through the point A (1, 3, 3) and meets P at (3, 1, 2). The acute angle between the plane P and the line l is θ. Find θ to the nearest degree $\theta = 63°$ (nearest degree)

Worked example	Your turn
Find the acute angle between the planes: Π_1: $\boldsymbol{r} \cdot (3\boldsymbol{i} - \boldsymbol{j} + \boldsymbol{k}) = 4$ Π_2: $\boldsymbol{r} \cdot (2\boldsymbol{i} + 3\boldsymbol{j}) = 7$	Find the acute angle between the planes: Π_1: $\boldsymbol{r} \cdot (4\boldsymbol{i} + 4\boldsymbol{j} - 7\boldsymbol{k}) = 13$ Π_2: $\boldsymbol{r} \cdot (7\boldsymbol{i} - 4\boldsymbol{j} + 4\boldsymbol{k}) = 6$ $\theta = 78.6°$ (1 dp)

Worked example	Your turn
The lines l_1 and l_2 have Cartesian equations $\frac{x+6}{-1}=\frac{y-3}{2}=\frac{z-2}{3}$ and $\frac{x+8}{-2}=\frac{y-4}{3}=\frac{z+13}{-1}$ respectively. (a) Show that the point $A\ (-2,-5,-10)$ lies on both lines (b) Find the size of the acute angle between the lines at A	The lines l_1 and l_2 have Cartesian equations $\frac{x-6}{-1}=\frac{y+3}{2}=\frac{z+2}{3}$ and $\frac{x+5}{2}=\frac{y-15}{-3}=\frac{z-3}{1}$ respectively. (a) Show that the point $A\ (3,3,7)$ lies on both lines (b) Find the size of the acute angle between the lines at A (a) Shown (b) $69.1°$ (1 dp)

9.5) Points of intersection

Worked example	Your turn
The lines l_1 and l_2 have vector equations $r = i + 2j + 3k + \lambda(4i - 3j - 2k)$ and $r = -3i - 2k + \mu(-4i + 8j + 9k)$ respectively. Show that the two lines intersect, and find the position vector of the point of intersection.	The lines l_1 and l_2 have vector equations $r = 3i + j + k + \lambda(i - 2j - k)$ and $r = -2j + 3k + \mu(-5i + j + 4k)$ respectively. Show that the two lines intersect, and find the position vector of the point of intersection. $(5, -3, -1)$

Worked example	Your turn
Find the point of intersection of the line l and the plane Π where: l: $\boldsymbol{r} = -\boldsymbol{2i} + \boldsymbol{j} + \boldsymbol{3k} + \lambda(2\boldsymbol{i} - \boldsymbol{j} + 7\boldsymbol{k})$ Π: $\boldsymbol{r} \cdot (3\boldsymbol{i} - 2\boldsymbol{j} + \boldsymbol{k}) = 5$	Find the point of intersection of the line l and the plane Π where: l: $\boldsymbol{r} = -\boldsymbol{i} + \boldsymbol{j} - 5\boldsymbol{k} + \lambda(\boldsymbol{i} + \boldsymbol{j} + 2\boldsymbol{k})$ Π: $\boldsymbol{r} \cdot (\boldsymbol{i} + 2\boldsymbol{j} + 3\boldsymbol{k}) = 4$ $(1, 3, -1)$

Worked example	Your turn
A line has Cartesian equation $$\frac{x-3}{-5}=\frac{y+2}{-3}=\frac{4-z}{-1}$$ A plane Π has Cartesian equation $$-4x-3y+2z=10$$ Find the position vector of the point of intersection of the line and the plane.	A line has Cartesian equation $$\frac{x-3}{5}=\frac{y+2}{3}=\frac{4-z}{1}$$ A plane Π has Cartesian equation $$4x+3y-2z=-10$$ Find the position vector of the point of intersection of the line and the plane. $$\left(\frac{53}{31},-\frac{86}{31},\frac{132}{32}\right)$$

Worked example	Your turn
The lines l_1 and l_2 have equations $\frac{x-3}{2} = y - 4 = \frac{z-1}{-5}$ and $\frac{x-1}{3} = \frac{y+2}{-7} = z - 5$ respectively. Prove that l_1 and l_2 are skew.	The lines l_1 and l_2 have equations $\frac{x-2}{4} = \frac{y+3}{2} = z - 1$ and $\frac{x+1}{5} = \frac{y}{4} = \frac{z-4}{-2}$ respectively. Prove that l_1 and l_2 are skew. Proof

9.6) Finding perpendiculars

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Worked example	Your turn
Find the shortest distance between the parallel lines with equations: $\boldsymbol{r} = 2\boldsymbol{i} - \boldsymbol{j} + 6\boldsymbol{k} + \lambda(3\boldsymbol{i} + 4\boldsymbol{j} + 5\boldsymbol{k})$ and $\boldsymbol{r} = 3\boldsymbol{j} - \boldsymbol{k} + \mu(6\boldsymbol{i} + 8\boldsymbol{j} + 10\boldsymbol{k})$, Where λ and μ are scalars	Find the shortest distance between the parallel lines with equations: $\boldsymbol{r} = \boldsymbol{i} + 2\boldsymbol{j} - \boldsymbol{k} + \lambda(5\boldsymbol{i} + 4\boldsymbol{j} + 3\boldsymbol{k})$ and $\boldsymbol{r} = 2\boldsymbol{i} + \boldsymbol{k} + \mu(5\boldsymbol{i} + 4\boldsymbol{j} + 3\boldsymbol{k})$, Where λ and μ are scalars $\frac{21\sqrt{2}}{10}$

Worked example	Your turn
The lines l_1 and l_2 have equations $r = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$ and $r = \begin{pmatrix} 1 \\ 3 \\ -1 \end{pmatrix} + \mu \begin{pmatrix} -1 \\ -1 \\ 2 \end{pmatrix}$ respectively, where λ and μ are scalars. Find the shortest distance between these two lines.	The lines l_1 and l_2 have equations $r = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$ and $r = \begin{pmatrix} -1 \\ 3 \\ -1 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ -1 \\ -1 \end{pmatrix}$ respectively, where λ and μ are scalars. Find the shortest distance between these two lines. $2\sqrt{2}$

Worked example

The line l has equation $\frac{x-1}{-2} = \frac{y-1}{2} = \frac{z+3}{1}$, and the point A has coordinates $(-1,2,1)$.

(a) Find the shortest distance between A and l.

(b) Find the Cartesian equation of the line that is perpendicular to l and passes through A.

Your turn

The line l has equation $\frac{x-1}{2} = \frac{y-1}{-2} = \frac{z+3}{-1}$, and the point A has coordinates $(1,2,-1)$.

(a) Find the shortest distance between A and l.

(b) Find the Cartesian equation of the line that is perpendicular to l and passes through A.

(a) $\frac{\sqrt{29}}{3}$

(b) $\frac{x-1}{8} = \frac{y-2}{1} = \frac{z+1}{14}$

Worked example	Your turn
Find the perpendicular distance from the point with coordinates $(1, -2, 3)$ to the plane with equation $3x - 2y - z = 5$.	Find the perpendicular distance from the point with coordinates $(3,2,-1)$ to the plane with equation $2x - 3y + z = 5$. $\frac{6}{\sqrt{14}}$

Worked example	Your turn
The plane Π has vector equation $$\boldsymbol{r}\cdot(2\boldsymbol{i}-3\boldsymbol{j}-\boldsymbol{k})=5$$ Find the perpendicular distance from the point $(2,-12,-23)$ to the plane	The plane Π has vector equation $$\boldsymbol{r}\cdot(3\boldsymbol{i}-4\boldsymbol{j}+2\boldsymbol{k})=5$$ Find the perpendicular distance from the point $(6,2,12)$ to the plane $\sqrt{29}$

Worked example	Your turn
The plane Π has equation $r \cdot (2i + 2j + 1k) = 5$. The point P has coordinates $(-2,3,1)$. (a) Find the shortest distance between P and Π.	The plane Π has equation $r \cdot (i + 2j + 2k) = 5$. The point P has coordinates $(1,3,-2)$. (a) Find the shortest distance between P and Π. $\frac{2}{3}$

Worked example	Your turn
The plane Π has equation $r \cdot (2i + 2j + 1k) = 5$. The point P has coordinates $(-2,3,1)$. The point Q is the reflection of the point P in Π. Find the coordinates of point Q.	The plane Π has equation $r \cdot (i + 2j + 2k) = 5$. The point P has coordinates $(1,3,-2)$. The point Q is the reflection of the point P in Π. Find the coordinates of point Q. $\left(\frac{12}{9}, \frac{35}{9}, -\frac{10}{9}\right)$

Worked example	Your turn
The line l_1 has equation $\frac{x-2}{-2}=\frac{y-4}{2}=\frac{z+6}{-1}$. The plane Π has equation $x-3y+2z=8$. The line l_2 is the reflection of line l_1 in the plane Π. Find a vector equation of the line l_2.	The line l_1 has equation $\frac{x-2}{2}=\frac{y-4}{-2}=\frac{z+6}{1}$. The plane Π has equation $2x-3y+z=8$. The line l_2 is the reflection of line l_1 in the plane Π. Find a vector equation of the line l_2. $\begin{pmatrix}6\\0\\-4\end{pmatrix}+t\begin{pmatrix}8\\-19\\4\end{pmatrix}$