Worked example	Your turn
A particle P of mass 2 kg moves in a horizontal straight line. At time t seconds, the displacement of P from a fixed point, O, on the line is x m and the velocity of P is v ms^{-1}. A force of magnitude $64x$ N acts on P in the direction PO. The particle is also subject to a resistance of magnitude $16v$ N. When $t = 0, x = 3$ and P is moving in the direction of increasing x with speed 1 ms^{-1}, (a) Show that $\frac{d^2x}{dt^2} + 16\frac{dx}{dt} + 64x = 0$ (b) Find the value of x when $t = 1$.	A particle P of mass 2 kg moves in a horizontal straight line. At time t seconds, the displacement of P from a fixed point, O, on the line is x m and the velocity of P is v ms^{-1}. A force of magnitude $8x$ N acts on P in the direction PO. The particle is also subject to a resistance of magnitude $4v$ N. When $t = 0, x = 1.5$ and P is moving in the direction of increasing x with speed 4 ms^{-1}, (a) Show that $\frac{d^2x}{dt^2} + 8\frac{dx}{dt} + 16x = 0$ (b) Find the value of x when $t = 1$. (a) Shown (b) $x = 0.211$ (3 sf)

Worked example

A particle P hangs freely in equilibrium attached to one end of a light elastic string. The other end of the string is attached to a fixed point A. The particle is now pulled down and held at rest in a container of liquid which exerts a resistance to motion on P. P is then released from rest. While the string remains taut and the particle in the liquid, the motion can be modelled using the equation $\frac{d^2x}{dt^2} + 4k\frac{dx}{dt} + 3k^2x = 0$, where k is a positive real constant
Find the general solution to the differential equation and state the type of damping that the particle is subject to.

Your turn

A particle P hangs freely in equilibrium attached to one end of a light elastic string. The other end of the string is attached to a fixed point A. The particle is now pulled down and held at rest in a container of liquid which exerts a resistance to motion on P. P is then released from rest. While the string remains taut and the particle in the liquid, the motion can be modelled using the equation $\frac{d^2x}{dt^2} + 6k\frac{dx}{dt} + 5k^2x = 0$, where k is a positive real constant
Find the general solution to the differential equation and state the type of damping that the particle is subject to.

$$x = Ae^{-5kt} + Be^{-kt}$$

Heavy damping

Worked example

One end of a light elastic spring is attached to a fixed point A. A particle P is attached to the other end and hangs in equilibrium vertically below A. At time $t = 0$, P is projected vertically downwards with speed u. A resistance proportional to the speed of P acts on P. The equation of motion of P is given as

$$\frac{d^2x}{dt^2} + 8k\frac{dx}{dt} + 16k^2x = 0$$

where k is a positive real constant and x is the displacement of P from its equilibrium position.

(a) Find the general solution to the differential equation.
(b) Find the time at which P comes to instantaneous rest.
(c) State the type of damping that the particle is subject to

Your turn

One end of a light elastic spring is attached to a fixed point A. A particle P is attached to the other end and hangs in equilibrium vertically below A. At time $t = 0$, P is projected vertically downwards with speed u. A resistance proportional to the speed of P acts on P. The equation of motion of P is given as

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where k is a positive real constant and x is the displacement of P from its equilibrium position.

(a) Find the general solution to the differential equation.
(b) Find the time at which P comes to instantaneous rest.
(c) State the type of damping that the particle is subject to

(a) $x = ute^{-2kt}$
(b) After $\frac{1}{2k}$ seconds
(c) Critical damping

Worked example

One end of a light elastic spring is attached to a fixed point A. A particle P is attached to the other end and hangs in equilibrium vertically below A. The particle is pulled vertically down from its equilibrium position and released from rest. A resistance proportional to the speed of P acts on P. The equation of motion of P is given as

$$\frac{d^2x}{dt^2} + 4k\frac{dx}{dt} + 8k^2x = 0$$

where k is a positive real constant and x is the displacement of P from its equilibrium position.

(a) Find the general solution to the differential equation.

(b) Write down the period of oscillation in terms of k.

(c) State the type of damping that the particle is subject to

Your turn

One end of a light elastic spring is attached to a fixed point A. A particle P is attached to the other end and hangs in equilibrium vertically below A. The particle is pulled vertically down from its equilibrium position and released from rest. A resistance proportional to the speed of P acts on P. The equation of motion of P is given as

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where k is a positive real constant and x is the displacement of P from its equilibrium position.

(a) Find the general solution to the differential equation.

(b) Write down the period of oscillation in terms of k.

(c) State the type of damping that the particle is subject to

(a) $x = e^{-kt}(A\cos kt + B\sin kt)$

(b) $\frac{2\pi}{k}$

(c) Light damping

Worked example	Your turn
A particle P of mass 1.5 kg is moving on the x-axis. At time t the displacement of P from the origin O is x metres and the speed of P is v ms^{-1}. Three forces act on P, namely a restoring force of magnitude $7.5x$ N, a resistance to the motion of P of magnitude $3v$ N and a force of magnitude $6\sin t$ N acting in the direction OP. When $t = 0, x = 5$ and $\frac{dx}{dt} = 2$. (a) Show that $\frac{dx^2}{dt^2} + 2\frac{dx}{dt} + 5x = 4\sin t$ (b) Find x as a function of t. (c) Describe the motion when t is large.	A particle P of mass 1.5 kg is moving on the x-axis. At time t the displacement of P from the origin O is x metres and the speed of P is v ms^{-1}. Three forces act on P, namely a restoring force of magnitude $7.5x$ N, a resistance to the motion of P of magnitude $6v$ N and a force of magnitude $12\sin t$ N acting in the direction OP. When $t = 0, x = 5$ and $\frac{dx}{dt} = 2$. (a) Show that $\frac{dx^2}{dt^2} + 4\frac{dx}{dt} + 5x = 8\sin t$ (b) Find x as a function of t. (c) Describe the motion when t is large. (a) Shown (b) $x = e^{-2t}(6\cos t + 13\sin t) + \sin t - \cos t$ (c) Simple harmonic motion with amplitude $\sqrt{2}$ and period 2π

Worked example

A particle P is attached to end A of a light elastic spring AB. Initially the particle and the string lie at rest on a smooth horizontal plane. At time $t = 0$, the end B of the string is set in motion and moves with constant speed U in the direction AB, and the displacement of P from A is x. Air resistance acting on P is proportional to its speed. The subsequent motion can be modelled by the differential equation

$$\frac{d^2x}{dt^2} + 4k\frac{dx}{dt} + 4k^2x = 2kU$$

Find an expression for x in terms of U, k and t

Your turn

A particle P is attached to end A of a light elastic spring AB. Initially the particle and the string lie at rest on a smooth horizontal plane. At time $t = 0$, the end B of the string is set in motion and moves with constant speed U in the direction AB, and the displacement of P from A is x. Air resistance acting on P is proportional to its speed. The subsequent motion can be modelled by the differential equation

$$\frac{d^2x}{dt^2} + 2k\frac{dx}{dt} + k^2x = 2kU$$

Find an expression for x in terms of U, k and t

$$x = \left(-\frac{2U}{k} - Ut\right)e^{-kt} + \frac{2U}{k}$$