8.3) Damped and forced harmonic motion

Worked example

A particle P is moving in a straight line.
At time t, the displacement of P from a fixed point on the line is x.
The motion of the particle is modelled by the differential equation

$$\frac{d^2x}{dt^2} + 6\frac{dx}{dt} + 5x = 0$$

When $t = 0, P$ is at rest at the point where $x = 3$

(a) Find x as a function of t

(b) Calculate the value of x when $t = \frac{2\pi}{3}$

(c) State whether the motion is heavily, critically or lightly damped

Your turn

A particle P is moving in a straight line.
At time t, the displacement of P from a fixed point on the line is x.
The motion of the particle is modelled by the differential equation

$$\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + 3x = 0$$

When $t = 0, P$ is at rest at the point where $x = 2$

(a) Find x as a function of t

(b) Calculate the value of x when $t = \frac{\pi}{3}$

(c) State whether the motion is heavily, critically or lightly damped

(a) $x = 3e^{-t} - e^{-3t}$
(b) 1.01 (3 sf)
(c) Heavily damped

Worked example

A particle P is moving in a straight line.
At time t, the displacement of P from a fixed point on the line is x.
The motion of the particle is modelled by the differential equation

$$\frac{d^2x}{dt^2} + 6\frac{dx}{dt} + 9x = 0$$

When $t = 0, P$ is at rest at the point where $x = 3$

(a) Find x as a function of t

(b) Calculate the value of x when $t = \frac{2\pi}{3}$

(c) State whether the motion is heavily, critically or lightly damped

Your turn

A particle P is moving in a straight line.
At time t, the displacement of P from a fixed point on the line is x.
The motion of the particle is modelled by the differential equation

$$\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + 4x = 0$$

When $t = 0, P$ is at rest at the point where $x = 2$

(a) Find x as a function of t

(b) Calculate the value of x when $t = \frac{\pi}{3}$

(c) State whether the motion is heavily, critically or lightly damped

(a) $x = (2 + 4t)e^{-2t}$
(b) 0.762 (3 sf)
(c) Critically damped

Worked example

A particle P is moving in a straight line.
At time t, the displacement of P from a fixed point on the line is x.
The motion of the particle is modelled by the differential equation

$$\frac{d^2x}{dt^2} + 6\frac{dx}{dt} + 18x = 0$$

When $t = 0, P$ is at rest at the point where $x = 3$

(a) Find x as a function of t

(b) Calculate the value of x when $t = \frac{2\pi}{3}$

(c) State whether the motion is heavily, critically or lightly damped

Your turn

A particle P is moving in a straight line.
At time t, the displacement of P from a fixed point on the line is x.
The motion of the particle is modelled by the differential equation

$$\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + 8x = 0$$

When $t = 0, P$ is at rest at the point where $x = 2$

(a) Find x as a function of t

(b) Calculate the value of x when $t = \frac{\pi}{3}$

(c) State whether the motion is heavily, critically or lightly damped

(a) $x = 2e^{-2t}(\cos 2t + \sin 2t)$
(b) 0.0901 (3 sf)
(c) Lightly damped