Type 3: Matrix Proofs

Example

Prove by induction that $\begin{pmatrix} 1 & -1 \\ 0 & 2 \end{pmatrix}^n = \begin{pmatrix} 1 & 1-2^n \\ 0 & 2^n \end{pmatrix}$ for all $n \in \mathbb{Z}^+$.

Example

Prove by induction that $\begin{pmatrix} -2 & 9 \\ -1 & 4 \end{pmatrix}^n = \begin{pmatrix} -3n+1 & 9n \\ -n & 3n+1 \end{pmatrix}$ for all $n \in \mathbb{Z}^+$.

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Be the examiner!

How many marks does this deserve??

6. Prove by induction that for all positive integers n

$$f(n) = 3^{2n+4} - 2^{2n}$$

is divisible by 5

(6)

Let ~~f~~ $n = 1$:

$f(1) = 3^{2(1)+4} - 2^{2(1)} = 3^6 - 2^2 = 729 - 4 = 725 = 145 \times 5$

$\therefore$ $f(n)$ divisible by 5 when $n = 1$

Assume true for $n = k$:

$5 \mid f(k) \Rightarrow f(k) = 3^{2k+4} - 2^{2k} = 5a$ where $a \in \mathbb{N}$

Consider $f(k+1)$:

$f(k+1) = 3^{2(k+1)+4} - 2^{2(k+1)} = 3^{2k+6} - 2^{2k+2} = 9(3^{2k+4}) - 4(2^{2k})$

$\Rightarrow f(k+1) = 9(3^{2k+4} - 2^{2k}) + 5(2^{2k}) = 9f(k) + 5(2^{2k}) = 5(9a + 2^{2k})$

~~f(k+1) is divisible by 5~~

$\therefore$ $f(k)$ divisible by 5 $\Rightarrow$ $f(k+1)$ divisible by 5

Since true for $n=1$ $\Rightarrow$ true for $\forall n \in \mathbb{Z}^+$ by mathematical induction.