8.2) Simple harmonic motion

Worked example

A particle is moving along a straight line.
At time t seconds its displacement, x m from a fixed point O is such that $\frac{d^2x}{dt^2} = -9x$.
Given that at $t = 0, x = 2$ and the particle is moving with velocity 9 ms^{-1},
(a) find an expression for the displacement of the particle after t seconds
(b) hence determine the maximum displacement of the particle from O.

Your turn

A particle is moving along a straight line.
At time t seconds its displacement, x m from a fixed point O is such that $\frac{d^2x}{dt^2} = -4x$.
Given that at $t = 0, x = 1$ and the particle is moving with velocity 4 ms^{-1},
(a) find an expression for the displacement of the particle after t seconds
(b) hence determine the maximum displacement of the particle from O.

(a) $x = \cos 2t + 2\sin 2t$
(b) $\sqrt{5}$

Worked example

A particle P, is attached to the ends of two identical elastic springs. The free ends of the springs are attached to two points A and B. The point C lies between A and B such that ABC is a straight line and $AC \neq BC$. The particle is held at C and then released from rest.

At time t seconds, the displacement of the particle from C is x m and its velocity is v ms^{-1}.

The subsequent motion of the particle can be described by the differential equation $\ddot{x} = -16x$.

(a) Describe the motion of the particle.

Given that $x = 0.5$ and $v = 0$ when $t = 0$,

(b) solve the differential equation to find x as a function of t

(c) state the period of the motion and calculate the maximum speed of P.

Your turn

A particle P, is attached to the ends of two identical elastic springs. The free ends of the springs are attached to two points A and B. The point C lies between A and B such that ABC is a straight line and $AC \neq BC$. The particle is held at C and then released from rest.

At time t seconds, the displacement of the particle from C is x m and its velocity is v ms^{-1}.

The subsequent motion of the particle can be described by the differential equation $\ddot{x} = -25x$.

(a) Describe the motion of the particle.

Given that $x = 0.4$ and $v = 0$ when $t = 0$,

(b) solve the differential equation to find x as a function of t

(c) state the period of the motion and calculate the maximum speed of P.

(a) Simple harmonic motion

(b) $x = 0.4 \cos 5t$

(c) Period $\frac{2\pi}{5}$ seconds. Max speed $2\ ms^{-1}$