8.1) Modelling with first-order differential equations

Worked example	Your turn
A particle P starts from rest at a point O and moves along a straight line. At time t seconds the acceleration, a ms^{-2}, of P is given by $$a = \frac{8}{(t+4)^2}, \qquad t \geq 0$$ (a) Find the velocity of P at time t seconds. (b) Find the displacement of P from O when $t = 8$	A particle P starts from rest at a point O and moves along a straight line. At time t seconds the acceleration, a ms^{-2}, of P is given by $$a = \frac{6}{(t+2)^2}, \qquad t \geq 0$$ (a) Find the velocity of P at time t seconds. (b) Find the displacement of P from O when $t = 6$ (a) $\left(3 - \frac{6}{t+2}\right) ms^{-1}$ (b) $(18 - 2\ln 2)\ m$

Worked example	Your turn
A particle P is moving along a straight line. At time t seconds, the acceleration of the particle is given by $a = t + \frac{6}{t}v,\ t \geq 0$ Given that $v = 0$ when $t = 4$, find the velocity of the particle at time t	A particle P is moving along a straight line. At time t seconds, the acceleration of the particle is given by $a = t + \frac{3}{t}v,\ t \geq 0$ Given that $v = 0$ when $t = 2$, find the velocity of the particle at time t $v = \frac{1}{2}t^3 - t^2$

Worked example

A fluid reservoir initially containers 10000 litres of unpolluted fluid.
The reservoir is leaking at a constant rate of 200 litres per hour and it is suspected that contaminated fluid flows into the reservoir at a constant rate of 200 litres per day.
The contaminated fluid contains 4 grams of contaminant in every litre of fluid.
It is assumed that the contaminant instantly disperses throughout the reservoir upon entry.
Given that there are x grams of contaminant in the reservoir after t days,

(a) Show that the situation can be modelled by the differential equation

$$\frac{dx}{dt} = 1200 - \frac{2x}{100 + t}$$

(b) Hence find the number of grams of contaminant in the tank after 7 days.

(c) Explain how the model could be refined.

Your turn

A storage tank initially containers 1000 litres of pure water.
Liquid is removed from the tank at a constant rate of 30 litres per hour and a chemical solution is added to the tank at a constant rate of 40 litres per hour.
The chemical solution contains 4 grams of copper sulphate per litre of water.
It is assumed that the copper suphate instantly disperses throughout the tank on entry.
Given that there are x grams of copper sulphate in the tank after t hours,

(a) Show that the situation can be modelled by the differential equation

$$\frac{dx}{dt} = 160 - \frac{3x}{100 + t}$$

(b) Hence find the number of grams of copper sulphate in the tank after 6 hours.

(c) Explain how the model could be refined.

(a) Shown

(b) 882 g (3 sf)

(c) The model could be refined to take into account the fact that the cupper sulphate does not disperse immediately on entering the tank.