8A Introduction to Proof by Induction

1. Prove by mathematical induction that, for $n \in \mathbb{Z}^+$

$$\sum_{r=1}^{n} (2r - 1) = n^2$$

2. Prove, by mathematical induction, that for $n \in \mathbb{Z}^+$,

$$\sum_{r=1}^{n} (r^2) = \frac{1}{6}n(n+1)(2n+1)$$

3. Prove, by mathematical induction, that for $n \in \mathbb{Z}^+$,

$$\sum_{r=1}^{n} (r2^{r}) = 2[1 + (n-1)2^{n}]$$