8A Introduction to Proof by Induction

1. Prove by mathematical induction that, for $n \in \mathbb{Z}^+$

$$\sum_{r=1}^{n} (2r - 1) = n^2$$

2. Prove, by mathematical induction, that for $n \in \mathbb{Z}^+$,

$$\sum_{r=1}^{n}(r^2) = \frac{1}{6}n(n+1)(2n+1)$$

3. Prove, by mathematical induction, that for $n \in \mathbb{Z}^+$,

$$\sum_{r=1}^{n} (r2^r) = 2[1 + (n-1)2^n]$$

8B Divisibility Proof By Induction

1. Prove, by induction, that $3^{2n} + 11$ is divisible by 4 for all positive integers $n \in \mathbb{Z}^+$

2. Prove, by induction, that the expression '$n^3 - 7n + 9$' is divisible by 3 for all positive integers $n \in \mathbb{Z}^+$

3. Prove, by induction, that the expression '$11^{n+1} + 12^{2n-1}$' is divisible by 133 for all positive integers $n \in \mathbb{Z}^+$

5C Composite Volumes of Revolution

1. Use mathematical induction to prove that:

$$\begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix}^n = \begin{bmatrix} 1 & 1-2^n \\ 0 & 2^n \end{bmatrix} \; for \; n \in \mathbb{Z}^+$$

2. Use mathematical induction to prove that:

$$\begin{bmatrix} -2 & 9 \\ -1 & 4 \end{bmatrix}^n = \begin{bmatrix} -3n+1 & 9n \\ -n & 3n+1 \end{bmatrix} \; for \; n \in \mathbb{Z}^+$$