**8A Introduction to Proof by Induction**

1. Prove by mathematical induction that, for $n\in Z^{+}$

$$\sum\_{r=1}^{n}\left(2r-1\right)=n^{2}$$

1. Prove, by mathematical induction, that for $n\in Z^{+}$,

$$\sum\_{r=1}^{n}\left(r^{2}\right)=\frac{1}{6}n\left(n+1\right)\left(2n+1\right)$$

1. Prove, by mathematical induction, that for $n\in Z^{+}$,

$$\sum\_{r=1}^{n}\left(r2^{r}\right)=2\left[1+(n-1)2^{n}\right]$$

**8B Divisibility Proof By Induction**

1. Prove, by induction, that 32n + 11 is divisible by 4 for all positive integers $n\in Z^{+}$
2. Prove, by induction, that the expression ‘n3 – 7n + 9’ is divisible by 3 for all positive integers $n\in Z^{+}$
3. Prove, by induction, that the expression ’11n+1 + 122n-1’ is divisible by 133 for all positive integers $n\in Z^{+}$

**5C Composite Volumes of Revolution**

1. Use mathematical induction to prove that:

$$\left[\begin{matrix}1&-1\\0&2\end{matrix}\right]^{n}=\left[\begin{matrix}1&1-2^{n}\\0&2^{n}\end{matrix}\right] for  n\in Z^{+}$$

1. Use mathematical induction to prove that:

$$\left[\begin{matrix}-2&9\\-1&4\end{matrix}\right]^{n}=\left[\begin{matrix}-3n+1&9n\\-n&3n+1\end{matrix}\right] for  n\in Z^{+}$$