8) Proof by induction

8.1) Proof by mathematical induction

Chapter CONTENTS

Worked example

Prove by induction that for all positive integers n:

$$\sum_{r=1}^{n} r(3r-1) = n^2(n+1)$$

Your turn

Prove by induction that for all positive integers n:

$$\sum_{r=1}^{n} (2r-1) = n^2$$

Proof

Worked example

Prove by induction that for all positive integers n:

$$\sum_{r=1}^{n} r^3 = \frac{1}{4}n^2(n+1)^2$$

Your turn

Prove by induction that for all positive integers n:

$$\sum_{r=1}^{n} r^2 = \frac{1}{6}n(n+1)(2n+1)$$

Proof

Worked example

Prove by induction that for all positive integers n:

$$\sum_{r=1}^{n} r2^r = 2(1 + (n-1)2^n)$$

Your turn

Prove by induction that for all positive integers n:

$$\sum_{r=1}^{n} \left(\frac{1}{2}\right)^r = 1 - \frac{1}{2^n}$$

Proof

8.2) Proving divisibility results

Chapter CONTENTS

Worked example

Prove by induction that for all positive integers n:
$3^{2n} - 1$ is divisible by 8

Your turn

Prove by induction that for all positive integers n:
$3^{2n} + 11$ is divisible by 4

Proof

Worked example

Prove by induction that for all positive integers n:
$5^n + 9^n + 2$ is divisible by 4

Your turn

Prove by induction that for all positive integers n:
$7^n + 4^n + 1$ is divisible by 6

Proof

Worked example

Prove by induction that for all positive integers n:
$8^n - 3^n$ is divisible by 5

Your turn

Prove by induction that for all positive integers n:
$13^n - 6^n$ is divisible by 7

Proof

Worked example

Prove by induction that for all positive integers n:
$2^{6n} + 3^{2n-2}$ is divisible by 5

Your turn

Prove by induction that for all positive integers n:
$11^{n+1} + 12^{2n-1}$ is divisible by 133

Proof

Worked example

Prove by induction that for all positive integers n:
$n^3 + 6n^2 + 8n$ is divisible by 3

Your turn

Prove by induction that for all positive integers n:
$n^3 - 7n + 9$ is divisible by 3

Proof

8.3) Proving statements involving matrices

Chapter CONTENTS

Worked example

Prove by induction that for all positive integers n:

$$\begin{pmatrix} 9 & 16 \\ -4 & -7 \end{pmatrix}^n = \begin{pmatrix} 8n+1 & 16n \\ -4n & 1-8n \end{pmatrix}$$

Your turn

Prove by induction that for all positive integers n:

$$\begin{pmatrix} -2 & 9 \\ -1 & 4 \end{pmatrix}^n = \begin{pmatrix} -3n+1 & 9n \\ -n & 3n+1 \end{pmatrix}$$

Proof

Worked example

Prove by induction that for all positive integers n:

$$\begin{pmatrix} 2 & 0 \\ 1 & 1 \end{pmatrix}^n = \begin{pmatrix} 2^n & 0 \\ 2^n - 1 & 1 \end{pmatrix}$$

Your turn

Prove by induction that for all positive integers n:

$$\begin{pmatrix} 1 & -1 \\ 0 & 2 \end{pmatrix}^n = \begin{pmatrix} 1 & 1 - 2^n \\ 0 & 2^n \end{pmatrix}$$

Proof