7.6) The inverse of a linear transformation

Worked example

The triangle T has vertices at A, B and C.

The matrix $M = \begin{pmatrix} 1 & 3 \\ -1 & 4 \end{pmatrix}$ transforms T to the triangle T' with vertices at $A'(3,4)$, $B'(10,4)$ and $C'(-3,-4)$.

Determine the coordinates of A, B and C.

Your turn

The triangle T has vertices at A, B and C. The matrix $M = \begin{pmatrix} 4 & -1 \\ 3 & 1 \end{pmatrix}$ transforms T to the triangle T' with vertices at $A'(4,3)$, $B'(4,10)$ and $C'(-4,-3)$. Determine the coordinates of A, B and C.

$A\,(1,0)\;\; B(2,4)\;\; C(-1,0)$

Worked example

$$M = \begin{pmatrix} 5 & -2 \\ 4 & -3 \end{pmatrix}$$

$$A = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

a) Find det M

b) Describe fully the single geometrical transformation represented by A

c) The transformation represented by A followed by the transformation represented by B is equivalent to the transformation represented by M. Find B

Your turn

$$M = \begin{pmatrix} 3 & 4 \\ 2 & -5 \end{pmatrix}$$

$$A = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

a) Find det M

b) Describe fully the single geometrical transformation represented by A

c) The transformation represented by A followed by the transformation represented by B is equivalent to the transformation represented by M. Find B

a) -23

b) Rotation $90°$ anticlockwise about $(0, 0)$

c) $\begin{pmatrix} -4 & 3 \\ 5 & 2 \end{pmatrix}$

Worked example

$$M = \begin{pmatrix} 1 & 0 & 1 \\ 1 & 3 & -1 \\ 0 & 2 & -2 \end{pmatrix}$$

The point (a, b, c) is mapped onto $(-3, -2, 1)$ under M. Find the values of a, b and c

Your turn

$$M = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & -2 \\ 1 & 3 & -1 \end{pmatrix}$$

The point (a, b, c) is mapped onto $(3, 2, -1)$ under M. Find the values of a, b and c

$a = 10, b = -6, c = -7$

Worked example

$$R = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

a) Find R^{-1}
b) Explain this geometrically
c) Find R^{7999}
d) Find R^{8000}

Your turn

$$R = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$$

a) Find R^{-1}
b) Explain this geometrically
c) Find R^{8001}
d) Find R^{8002}

a) $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$
b) Reflection is self-inverse. The inverse of reflection in the line $y = -x$ is reflection in the line $y = -x$ again
c) $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$
d) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$