7.5) Linear transformations in three dimensions

Worked example

Find the matrix representing:

- reflection in the plane $x = 0$

- reflection in the plane $y = 0$

Your turn

Find the matrix representing:

- reflection in the plane $z = 0$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

Worked example

$$\mathbf{M} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

(a) Describe the transformation represented by $\mathbf{M}$.
(b) Find the image of the point with coordinates $(-1, 2, 3)$ under the transformation represented by $\mathbf{M}$.

Your turn

$$\mathbf{M} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

(a) Describe the transformation represented by $\mathbf{M}$.
(b) Find the image of the point with coordinates $(-1, 2, 3)$ under the transformation represented by $\mathbf{M}$.

(a) Reflection in the plane $z = 0$
(b) $(-1, 2, -3)$

Worked example

Find the matrix representing:

- Rotation, angle θ, anticlockwise about the x-axis

- Rotation, angle θ, anticlockwise about the y-axis

Your turn

Find the matrix representing:

- Rotation, angle θ, anticlockwise about the z-axis

$$\begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Worked example

Find the matrix representing:

- Rotation, angle $90°$, anticlockwise about the x-axis

- Rotation, angle $180°$, anticlockwise about the z-axis

Your turn

Find the matrix representing:

- Rotation, angle $270°$, anticlockwise about the y-axis

$$\begin{pmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

Worked example

$$\mathbf{M} = \begin{pmatrix} -\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 \\ \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

(a) Describe the transformation represented by **M**.
(b) Find the image of the point with coordinates $(-1, -2, 1)$ under the transformation represented by **M**.

Your turn

$$\mathbf{M} = \begin{pmatrix} \frac{\sqrt{3}}{2} & 0 & \frac{1}{2} \\ 0 & 1 & 0 \\ -\frac{1}{2} & 0 & \frac{\sqrt{3}}{2} \end{pmatrix}$$

(a) Describe the transformation represented by **M**.
(b) Find the image of the point with coordinates $(-1, -2, 1)$ under the transformation represented by **M**.

(a) Rotation 30° anticlockwise about the y-axis
(b) $\left(\frac{1-\sqrt{3}}{2}, -2, \frac{1+\sqrt{3}}{2}\right)$