

7.3) Second-order non-homogenous differential equations

Worked example

Find the general solution to:

$$\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 6y = 4$$

Your turn

Find the general solution to:

$$\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 3$$

$$y = Ae^{3x} + Be^{2x} + \frac{1}{2}$$

Worked example

Find the general solution to:

$$\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 6y = 4x$$

Your turn

Find the general solution to:

$$\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 3x$$

$$y = Ae^{3x} + Be^{2x} + \frac{1}{2}x + \frac{5}{12}$$

Worked example

Find the general solution to:

$$\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 6y = 4x^2$$

Your turn

Find the general solution to:

$$\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 3x^2$$

$$y = Ae^{3x} + Be^{2x} + \frac{1}{2}x^2 + \frac{5}{6}x + \frac{19}{36}$$

Worked example

Find the general solution to:

$$\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 6y = e^{-x}$$

Your turn

Find the general solution to:

$$\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = e^x$$

$$y = Ae^{3x} + Be^{2x} + \frac{1}{2}e^x$$

Worked example

Find the general solution to:

$$\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 6y = 7 \sin 4x$$

Your turn

Find the general solution to:

$$\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 13 \sin 3x$$

$$y = Ae^{3x} + Be^{2x} - \frac{1}{6} \sin 3x + \frac{5}{6} \cos 3x$$

Worked example

Find the general solution to:

$$\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 6y = e^{-3x}$$

Your turn

Find the general solution to:

$$\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = e^{2x}$$

$$y = Ae^{3x} + Be^{2x} - xe^{2x}$$

Worked example

Find the general solution to:

$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} = 2$$

Your turn

Find the general solution to:

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} = 3$$

$$y = A + Be^{2x} - \frac{3}{2}x$$

Worked example

Find the general solution to:

$$\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 4y = x^2 - 2x + 3$$

Your turn

Find the general solution to:

$$\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 4y = x^2 - 3x + 2$$

$$y = Ae^{4x} + Be^x + \frac{1}{4}x^2 - \frac{1}{8}x + \frac{7}{32}$$

Worked example

Find the general solution to:

$$\frac{d^2y}{dx^2} - 6\frac{dy}{dx} = 2x^2 - x + 1$$

Your turn

Find the general solution to:

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} = 24x^2$$

$$y = A + Be^{-4x} + 2x^3 - \frac{3}{2}x^2 + \frac{3}{4}x$$

Worked example

Find the general solution to:

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 1 = e^x$$

Your turn

Find the general solution to:

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 1 = e^x$$

$$y = \left(A + Bx + \frac{1}{2}x^2 \right) e^x$$

Worked example

Find the general solution to:

$$\frac{d^2x}{dt^2} - 5\frac{dx}{dt} + 6x = 2\sin t - \cos t$$

Your turn

Find the general solution to:

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = 2\cos t - \sin t$$

$$x = Ae^{-3t} + Be^{-3t} + \frac{3}{10}\cos t + \frac{1}{10}\sin t$$