7.1) Linear transformations in two dimensions

Worked example	Your turn
Find matrices to represent these linear transformations.	Find matrices to represent these linear transformations.
a) $T: \begin{pmatrix} x \\ y \end{pmatrix} \to \begin{pmatrix} 3y - x \\ 2x \end{pmatrix}$	a) $T: \begin{pmatrix} x \\ y \end{pmatrix} \to \begin{pmatrix} 2y + x \\ 3x \end{pmatrix}$ $\begin{pmatrix} 1 & 2 \\ 3 & 0 \end{pmatrix}$
b) $V: \begin{pmatrix} x \\ y \end{pmatrix} \to \begin{pmatrix} -y \\ x + 3y \end{pmatrix}$	b) $V: \begin{pmatrix} x \\ y \end{pmatrix} \to \begin{pmatrix} -2y \\ 3x + y \end{pmatrix}$ $\begin{pmatrix} 0 & -2 \\ 3 & 1 \end{pmatrix}$

Worked example	Your turn
A rectangle R has vertices $(2, 1), (4, 1), (4, 2)\ and\ (2, 2)$ Find the vertices of the image of R under the transformation given by the matrix $\boldsymbol{M} = \begin{pmatrix} 1 & 3 \\ 3 & -1 \end{pmatrix}$. Sketch R and its image, R' on a coordinate grid.	A square has vertices $(1,1), (3,1), (3,3)$ and $(1,3)$ Find the vertices of the image of S under the transformation given by the matrix $\boldsymbol{M} = \begin{pmatrix} -1 & 2 \\ 2 & 1 \end{pmatrix}$. Sketch S and the image of S on a coordinate grid. $(1,3), (-1,7), (3,9), (5,5)$

Worked example	Your turn
Determine if the point (2, 5) is invariant under the transformation given by the matrix: $$\begin{pmatrix} 1 & 6 \\ 4 & 3 \end{pmatrix}$$ $$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$	Determine if the point (4, 6) is invariant under the transformation given by the matrix: $$\begin{pmatrix} 2 & 1 \\ 3 & 5 \end{pmatrix}$$ No

Worked example	Your turn
Determine whether $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ has any lines of invariant points	Determine whether $\begin{pmatrix} 5 & -2 \\ 3 & -0.5 \end{pmatrix}$ has any lines of invariant points $y = 2x$

Worked example	Your turn
Show that the matrix $\begin{pmatrix} 2 & -5 \\ 4 & -3 \end{pmatrix}$ has no invariant points other than the origin	Show that the matrix $\begin{pmatrix} 4 & -3 \\ 2 & -5 \end{pmatrix}$ has no invariant points other than the origin

$$\begin{pmatrix} 4 & -3 \\ 2 & -5 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$$

$4x - 3y = x$ -> $y = x$

$2x - 5y = y$ -> $y = \frac{1}{3}x$

$x = \frac{1}{3}x$ -> $x = 0, y = 0$

$\therefore (0, 0)$ is the only invariant point

Worked example	Your turn
Find the invariant lines of the transformation given by $\begin{pmatrix} 6 & 5 \\ 2 & 3 \end{pmatrix}$	Find the invariant lines of the transformation given by $\begin{pmatrix} 3 & 2 \\ 5 & 6 \end{pmatrix}$ $y = \frac{5}{2}x + c$ $y = -x$