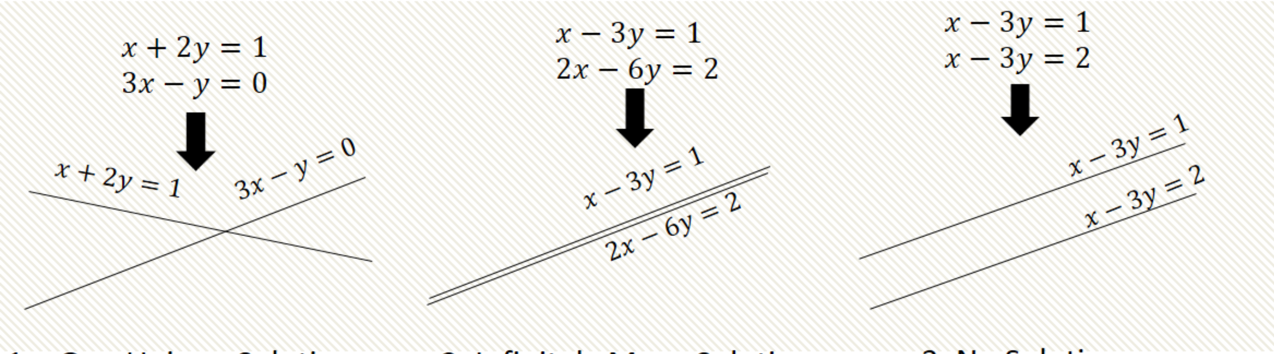
Consistency of Linear Equations

As we know, the solution of a system of two equations (with two unknowns) can be visualised by finding the point of intersection of two lines.
A system of linear equations is known as consistent if there is at least one set of values that satisfies all the equations simultaneously (i.e. at least one point of intersection).



1. One Unique Solution

The system of equations is **consistent.** It has **one solution.**

The corresponding matrix $\begin{pmatrix} 1 & 2 \\ 3 & -1 \end{pmatrix}$ is **non-singular.**

$|A| \neq 0$

2. Infinitely Many Solutions

The system of equations is **consistent.** It has **infinitely many solutions.**

Matrix $\begin{pmatrix} 1 & -3 \\ 1 & -3 \end{pmatrix}$ is **singular.**

$|A| = 0$

3. No Solution

The system of equations is **inconsistent.** It has **no solutions.**

Matrix $\begin{pmatrix} 1 & -3 \\ 1 & -3 \end{pmatrix}$ is **singular.**

$|A| = 0$

Extending to 3 Variables

Again, we get solutions to the system of linear equations when all of the planes intersect.

Consider the possible outcomes for a set of 3 planes:

Scenario 1

Scenario 2

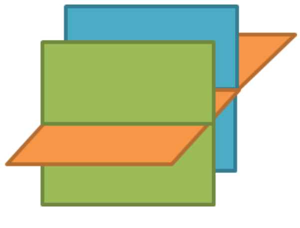
1.

2.

Scenario 3

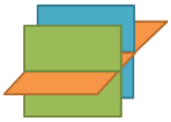
1.

2.



To classify solutions, we should:

1. First check for identical planes (equations which are equivalent) and therefore infinite solutions or parallel planes and therefore no solutions.



2. Next find the value of det A. If $|A| \neq 0$ the system of equations is consistent and there exists one unique solution.

3. If det A = 0 we have to check for parallel planes, either by sight (rows of matrix A are multiples) or by eliminating a variable and looking at the resulting linear equations…….

4. If the resulting 2d linear equations represent the same line then the original equations are consistent and therefore form a sheaf.

5. Otherwise, the planes form a prism and the system is inconsistent with no unique solution. (Parallel planes can be eliminated from the original equations)

Example

A system of equations is shown below:

$$3x - ky - 6z = k$$

$$kx + 3y + 3z = 2$$

$$-3x - y + 3z = -2$$

For each of the following values of k, determine whether the system of equations is consistent or inconsistent. If the system is consistent, determine whether there is a unique solution or an infinity of solutions. In each case, identify the geometric configuration of the plane corresponding to each value of k.

(a) $k = 0$ (b) $k = 1$ (c) $k = -6$

Test Your Understanding

The system of equations is consistent and has a single solution. Determine the possible values of k.

$$2x + 3y - z = 13$$

$$3x - y + kz = 11$$

$$x + y + z = 7$$

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