

## Worked example

A system of equations is shown below:

$$3x - ky - 6z = k$$

$$kx + 3y + 3z = 2$$

$$-3x - y + 3z = -2$$

For each of the following values of  $k$ , determine whether the system of equations is consistent or inconsistent.

If the system is consistent, determine whether there is a unique solution or an infinity of solutions.

In each case, identify the geometric configuration of the plane corresponding to each value of  $k$ .

(a)  $k = 0$

(b)  $k = -6$

## Your turn

A system of equations is shown below:

$$3x - ky - 6z = k$$

$$kx + 3y + 3z = 2$$

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For each of the following values of  $k$ , determine whether the system of equations is consistent or inconsistent.

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In each case, identify the geometric configuration of the plane corresponding to each value of  $k$ .

(a)  $k = 1$

$$(a) k = 1: \begin{vmatrix} 3 & -1 & -6 \\ 1 & 3 & 3 \\ -3 & -1 & 3 \end{vmatrix} = 0$$

$$3x - y - 6z = 1 \quad (1)$$

$$x + 3y + 3z = 2 \quad (2)$$

$$-3x - y + 3z = -2 \quad (3)$$

$$(1) + 2 \times (2): 5x + 5y = 5 \quad (4)$$

$$(2) - (3): 4x + 4y = 4 \quad (5)$$

Equations (4) and (5) are consistent so system is consistent and has an infinity of solutions. Planes meet at a sheaf

## Worked example

A system of equations is shown below:

$$\begin{aligned}x - ry - 6z &= r \\ rx - 4y - 12z &= s \\ -3x + ty + 18z &= u\end{aligned}$$

For each of the following values of  $r$ ,  $s$  and  $t$ , determine whether the system of equations is consistent or inconsistent.

If the system is consistent, determine whether there is a unique solution or an infinity of solutions.

In each case, identify the corresponding geometric configuration.

(a)  $r = 2, s = 5, t = 4, u = 1$

(b)  $r = 2, s = 4, t = 6, u = -6$

## Your turn

A system of equations is shown below:

$$\begin{aligned}x - ry - 6z &= r \\ rx - 4y - 12z &= s \\ -3x + ty + 18z &= u\end{aligned}$$

For each of the following values of  $r$ ,  $s$  and  $t$ , determine whether the system of equations is consistent or inconsistent.

If the system is consistent, determine whether there is a unique solution or an infinity of solutions.

In each case, identify the corresponding geometric configuration.

(a)  $r = 2, s = 4, t = 6, u = -5$

$$(a) \begin{vmatrix} 1 & -2 & -6 \\ 2 & -4 & -12 \\ -3 & 6 & 18 \end{vmatrix} = 0$$

$$x - 2y - 6z = 2 \quad (1)$$

$$2x - 4y - 12z = 4 \quad (2)$$

$$-3x + 6y + 18z = -5 \quad (3)$$

All three planes are parallel and non-identical.

The system of equations is inconsistent and has no solutions.