# 6.5) Inverting a 3 x 3 matrix

## Worked example

If $\boldsymbol{A} = \begin{pmatrix} 0 & -3 & -2 \\ 1 & -4 & -2 \\ -3 & 4 & 1 \end{pmatrix}$, find $\boldsymbol{A}^{-1}$.

## Your turn

If $\boldsymbol{A} = \begin{pmatrix} 1 & 3 & 1 \\ 0 & 4 & 1 \\ 2 & -1 & 0 \end{pmatrix}$, find $\boldsymbol{A}^{-1}$.

$$\begin{pmatrix} -1 & 1 & 1 \\ -2 & 2 & 1 \\ 8 & -7 & -4 \end{pmatrix}$$

## Worked example

$$\boldsymbol{A} = \begin{pmatrix} 5 & -4 & 4 \\ 8 & -7 & 8 \\ 2 & -2 & 3 \end{pmatrix},$$

Show that $\mathbf{A}^{-1} = \mathbf{A}$.

## Your turn

$$\boldsymbol{A} = \begin{pmatrix} -2 & 3 & -3 \\ 0 & 1 & 0 \\ 1 & -1 & 2 \end{pmatrix},$$

Show that $\mathbf{A}^{-1} = \mathbf{A}$.

$$A^2 = \begin{pmatrix} -2 & 3 & -3 \\ 0 & 1 & 0 \\ 1 & -1 & 2 \end{pmatrix}\begin{pmatrix} -2 & 3 & -3 \\ 0 & 1 & 0 \\ 1 & -1 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

## Worked example

$$\boldsymbol{A} = \begin{pmatrix} 5 & -4 & 4 \\ 8 & -7 & 8 \\ 2 & -2 & 3 \end{pmatrix},$$

The matrix $\mathbf{B}$ is such that $(\mathbf{AB})^{-1} = \begin{pmatrix} 2 & 5 & -3 \\ -4 & 1 & -8 \\ -1 & 0 & 11 \end{pmatrix}$.

Find $\boldsymbol{B}^{-1}$.

## Your turn

$$\boldsymbol{A} = \begin{pmatrix} -2 & 3 & -3 \\ 0 & 1 & 0 \\ 1 & -1 & 2 \end{pmatrix},$$

The matrix $\mathbf{B}$ is such that $(\mathbf{AB})^{-1} = \begin{pmatrix} 8 & -17 & 9 \\ -5 & 10 & -6 \\ -3 & 5 & -4 \end{pmatrix}$.

Find $\boldsymbol{B}^{-1}$.

$$(AB)^{-1} = B^{-1}A^{-1}$$
$$(AB)^{-1}A = B^{-1}A^{-1}A$$
$$(AB)^{-1}A = B^{-1}$$
$$B^{-1} = \begin{pmatrix} 8 & -17 & 9 \\ -5 & 10 & -6 \\ -3 & 5 & -4 \end{pmatrix}\begin{pmatrix} -2 & 3 & -3 \\ 0 & 1 & 0 \\ 1 & -1 & 2 \end{pmatrix}$$
$$= \begin{pmatrix} -7 & -2 & -6 \\ 4 & 1 & 3 \\ 2 & 0 & 1 \end{pmatrix}$$

## Worked example

$$\boldsymbol{A} = \begin{pmatrix} k & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -2 & 3 \end{pmatrix}, k \neq -1$$

Find the inverse matrix of A in terms of $k$

## Your turn

$$\boldsymbol{A} = \begin{pmatrix} k & -1 & 1 \\ 1 & 0 & -1 \\ 3 & -2 & 1 \end{pmatrix}, k \neq 1$$

Find the inverse matrix of A in terms of $k$

$$A^{-1} = \frac{1}{2-2k}\begin{pmatrix} -2 & -1 & 1 \\ -4 & k-3 & k+1 \\ -2 & 2k-3 & 1 \end{pmatrix}$$

## Worked example

Find the inverse of the matrix using elementary row operations

$$\begin{pmatrix} 3 & 0 & 2 \\ 2 & 0 & -2 \\ 0 & 1 & 1 \end{pmatrix}$$

## Your turn

Find the inverse of the matrix using elementary row operations

$$\begin{pmatrix} 1 & 0 & -3 \\ 2 & -2 & 1 \\ 0 & -1 & 3 \end{pmatrix}$$

$$\begin{pmatrix} -5 & 3 & -6 \\ -6 & 3 & -7 \\ -2 & 1 & -2 \end{pmatrix}$$