6.5) Integrating hyperbolic functions

Worked example	Your turn
Find:	Find:
$\int \cosh(3x-2)\,dx$	$\int \cosh(4x-1)\,dx$ — $\frac{1}{4}\sinh(4x-1)+c$
$\int sinh\left(\frac{5}{7}x\right)\,dx$	$\int sinh\left(\frac{2}{3}x\right)\,dx$ — $\frac{3}{2}\cosh\left(\frac{2}{3}x\right)+c$
$\int \frac{7}{\sqrt{1+x^2}}dx$	$\int \frac{3}{\sqrt{1+x^2}}dx$ — $3\,arsinh\,x+c$
$\int \frac{6}{\sqrt{x^2-1}}dx$	$\int \frac{4}{\sqrt{x^2-1}}dx$ — $4\,arcosh\,x+c$
$\int sinh\,(5x)\,\,dx$	$\int sinh\,(3x)\,\,dx$ — $\frac{1}{3}\cosh(3x)+c$
$\int \frac{4}{\sqrt{x^2-1}}dx$	$\int \frac{10}{\sqrt{x^2-1}}dx$ — $10\,arcosh\,x+c$
$\int \frac{3}{\sqrt{1+x^2}}dx$	$\int \frac{2}{\sqrt{1+x^2}}dx$ — $2\,arsinh\,x+c$

Worked example	Your turn
Find: $$\int \frac{3-7x}{\sqrt{x^2+1}}\,dx$$	Find: $$\int \frac{2+5x}{\sqrt{x^2+1}}\,dx$$ $$2\,arsinh\,x + 5\sqrt{1+x^2} + c$$

Worked example	Your turn
Find: $$\int \sinh^7 3x \cosh 3x \; dx$$	Find: $$\int \cosh^5 2x \sinh 2x \; dx$$ $$\frac{1}{12}\cosh^6 2x + c$$

Worked example	Your turn
Find: $$\int \coth x \ dx$$	Find: $$\int \tanh x \ dx$$ $$ln\lvert cosh\, x\rvert + C$$

Worked example	Your turn
Find: $$\int \sinh^2 5x \ dx$$	Find: $$\int \cosh^2 3x \ dx$$ $$\frac{1}{2}x + \frac{1}{12}\sinh 6x + c$$

Worked example	Your turn
Find: $$\int \cosh^3 x \ dx$$	Find: $$\int \sinh^3 x \ dx$$ $$\frac{1}{3}\cosh^3 x - \cosh x + c$$

Worked example	Your turn
Find: $$\int e^{3x} \cosh x \; dx$$	Find: $$\int e^{2x} \sinh x \; dx$$ $$\frac{1}{6}\left(e^{3x} - 3e^{x}\right) + c$$

Worked example	Your turn
Find: $$\int cosech\, x\, dx$$ (requires partial fractions)	Find: $$\int \operatorname{sech} x \;\; dx$$ $$2\arctan(e^{x}) + c$$

Worked example	Your turn
Show that $\int \frac{1}{\sqrt{a^2+x^2}}\,dx = arsinh\left(\frac{x}{a}\right) + c$	Show that $\int \frac{1}{\sqrt{x^2-a^2}}\,dx = arcosh\left(\frac{x}{a}\right) + c$ Shown

Worked example	Your turn
Evaluate $\int_5^8 \frac{1}{\sqrt{x^2+16}}\,dx$	Evaluate $\int_5^8 \frac{1}{\sqrt{x^2-16}}\,dx$ $\ln\left(\frac{2+\sqrt{3}}{2}\right)$

Worked example	Your turn
By using a hyperbolic substitution, show that $\int \sqrt{x^2-1}\,dx = \frac{1}{2}x\sqrt{x^2-1} - \frac{1}{2}arcosh\,x + c$	By using a hyperbolic substitution, show that $\int \sqrt{1+x^2}\,dx = \frac{1}{2}arsinh\,x + \frac{1}{2}x\sqrt{1+x^2} + c$ Shown using $x = \sinh u$ NB: Can also use $x = \tan u$ but longer

Worked example	Your turn
Using a hyperbolic substitution, evaluate $$\int_4^8 \frac{x^3}{\sqrt{x^2-16}}\,dx$$	Using a hyperbolic substitution, evaluate $$\int_0^6 \frac{x^3}{\sqrt{x^2+9}}\,dx$$ $18\sqrt{5}+18$

Worked example	Your turn
Find: $$\int \frac{1}{\sqrt{12x + 3x^2}}\,dx$$	Find: $$\int \frac{1}{\sqrt{12x + 2x^2}}\,dx$$ $$\frac{1}{\sqrt{2}} arcosh\left(\frac{x+3}{3}\right) + C$$

Worked example	Your turn
Find: $$\int \frac{1}{x^2 - 6x - 3}\,dx$$	Find: $$\int \frac{1}{x^2 - 8x + 8}\,dx$$ $$\frac{\sqrt{2}}{8}\ln\left|\frac{x-4-2\sqrt{2}}{x-4+2\sqrt{2}}\right| + C$$

Worked example	Your turn
Evaluate: $$\int_0^1 \frac{1}{\sqrt{x^2+8x+17}}\,dx$$	Evaluate: $$\int_0^1 \frac{1}{\sqrt{x^2+2x+5}}\,dx$$ 0.400 (3 sf)

Worked example	Your turn
Find: $$\int \frac{1}{\sqrt{9x^2+4}}\,dx$$	Find: $$\int \frac{1}{\sqrt{4x^2+9}}\,dx$$ $$\frac{1}{2}arsinh\left(\frac{2x}{3}\right)+c$$

Worked example	Your turn
$25x^2 + 10x + 17 \equiv a(x+b)^2 + c$	$9x^2 + 6x + 5 \equiv a(x+b)^2 + c$
a) Find the values of a, b and c	a) Find the values of a, b and c
Hence, or otherwise, find:	Hence, or otherwise, find:
b) $\int \frac{1}{25x^2+10x+17} dx$	b) $\int \frac{1}{9x^2+6x+5} dx$
c) $\int \frac{1}{\sqrt{25x^2+10x+17}} dx$	c) $\int \frac{1}{\sqrt{9x^2+6x+5}} dx$
	a) $a = 9, b = \frac{1}{3}, c = 4$
	b) $\frac{1}{6} \arctan \frac{3x+1}{2} + c$
	c) $\frac{1}{3} arsinh \frac{3x+1}{2} + c$