6.4) Inverting a 2 x 2 matrix

Worked example	Your turn
Find the inverse matrix for:	Find the inverse matrix for

$$\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$$

$$\begin{pmatrix} 4 & 3 \\ -2 & -1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & -2 \\ 3 & -4 \end{pmatrix}$$

$$\frac{1}{2}\begin{pmatrix} -4 & 2 \\ -3 & 1 \end{pmatrix} \text{ or } \begin{pmatrix} -2 & 1 \\ -\frac{3}{2} & \frac{1}{2} \end{pmatrix}$$

Worked example	Your turn
For what value of p is $\begin{pmatrix} 1 & 2-p \\ -4 & p+3 \end{pmatrix}$ singular?	For what value of p is $\begin{pmatrix} 4 & p+2 \\ -1 & 3-p \end{pmatrix}$ singular? $p = \frac{14}{3}$
Given p is not this value, find the inverse.	Given p is not this value, find the inverse. $\frac{1}{14-3p}\begin{pmatrix} 3-p & -(p+2) \\ 1 & 4 \end{pmatrix}$

Worked example	Your turn
If $\mathbf{A}$ and $\mathbf{B}$ are non-singular matrices, prove that $(\boldsymbol{AB})^{-1} = \boldsymbol{B}^{-1}\boldsymbol{A}^{-1}$	If $\mathbf{P}$ and $\mathbf{Q}$ are non-singular matrices, prove that $(\mathbf{PQ})^{-1} = \mathbf{Q}^{-1}\mathbf{P}^{-1}$

Let $C = (PQ)^{-1}$

$$(PQ)C = (PQ)(PQ)^{-1}$$
$$(PQ)C = I$$
$$P^{-1}PQC = P^{-1}I$$
$$IQC = P^{-1}$$
$$QC = P^{-1}$$
$$Q^{-1}QC = Q^{-1}P^{-1}$$
$$IC = Q^{-1}P^{-1}$$
$$C = Q^{-1}P^{-1}$$
$$(PQ)^{-1} = Q^{-1}P^{-1}$$

Worked example	Your turn
If A and B are non-singular matrices such that $\mathbf{ABA} = \mathbf{I}$, prove that $\mathbf{B} = \mathbf{A}^{-1}\mathbf{A}^{-1}$	If A and B are non-singular matrices such that $\mathbf{BAB} = \mathbf{I}$, prove that $\mathbf{A} = \mathbf{B}^{-1}\mathbf{B}^{-1}$

$$BAB = I$$
$$B^{-1}BAB = B^{-1}I$$
$$IAB = B^{-1}$$
$$AB = B^{-1}$$
$$ABB^{-1} = B^{-1}B^{-1}$$
$$AI = B^{-1}B^{-1}$$
$$A = B^{-1}B^{-1}$$