6.4) Differentiating hyperbolic functions

Worked example	Your turn
Prove that $\frac{d}{dx}(\sinh x) = \cosh x$	Prove that $\frac{d}{dx}(\cosh x) = \sinh x$ Proof

Worked example	Your turn
Differentiate with respect to x:	Differentiate with respect to x:
$\sinh 7x$	$\sinh 2x$
	$2\cosh 2x$
$\cosh 6x$	$\cosh 3x$
	$3\sinh 3x$
$\tanh 5x$	$\tanh 4x$
	$4\operatorname{sech}^2 4x$

Worked example	Your turn
Differentiate with respect to x: $x^3 \sinh 5x$	Differentiate with respect to x: $x^2 \cosh 4x$ $2x \cosh 4x + 4x^2 \sinh 4x$

Worked example	Your turn
$y = \frac{1}{2}\ln(\tanh x)$ Show that $\frac{dy}{dx} = cosech\ 2x$	$y = \frac{1}{2}\ln(\coth x)$ Show that $\frac{dy}{dx} = -cosech\ 2x$ Shown

Worked example	Your turn
Prove that $$\frac{d}{dx}(\text{arsinh}\, x) = \frac{1}{\sqrt{x^2+1}}$$	Prove that $$\frac{d}{dx}(\text{arcosh}\, x) = \frac{1}{\sqrt{x^2-1}}$$ $$y = arcosh\, x$$ $$x = \cosh y$$ $$\frac{dx}{dy} = \sinh y$$ $$\frac{dy}{dx} = \frac{1}{\sinh y}$$ $$\frac{dy}{dx} = \frac{1}{\sqrt{\cosh^2 y - 1}}$$ $$\frac{dy}{dx} = \frac{1}{\sqrt{x^2-1}}$$

Worked example	Your turn
Differentiate with respect to x:	Differentiate with respect to x:
$arsinh\ 7x$	$arsinh\ 2x$ $\dfrac{2}{\sqrt{4x^2+1}}$
$arcosh\ 6x$	$arcosh\ 3x$ $\dfrac{3}{\sqrt{9x^2-1}}$
$artanh\ 5x$	$artanh\ 4x$ $\dfrac{4}{1-16x^2}$

Worked example	Your turn
Given that $y = (arsinh\ x)^4$ prove that $(x^2 + 1)\left(\frac{dy}{dx}\right)^2 = 16y^{\frac{3}{2}}$	Given that $y = (arcosh\ x)^2$ prove that $(x^2 - 1)\left(\frac{dy}{dx}\right)^2 = 4y$ Proof

Worked example

(a) Show that $\frac{d}{dx}(arsinh\ x) = \frac{1}{\sqrt{1+x^2}}$

(b) Find the first two non-zero terms of the series expansion of $arsinh\ x$.

The general form for the series expansion of $arsinh\ x$ is given by

$$arsinh\ x = \sum_{r=0}^{\infty} \left(\frac{(-1)^n (2n)!}{2^{2n}(n!)^2} \right) \frac{x^{2n+1}}{2n+1}$$

(c) Find, in simplest terms, the coefficient of x^7.

(d) Use your approximation up to and including the term in x^7 to find an approximate value for $arsinh\ 0.5$.

(e) Calculate the percentage error in using this approximation.

Your turn

(a) Show that $\frac{d}{dx}(arsinh\ x) = \frac{1}{\sqrt{1+x^2}}$

(b) Find the first two non-zero terms of the series expansion of $arsinh\ x$.

The general form for the series expansion of $arsinh\ x$ is given by

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(c) Find, in simplest terms, the coefficient of x^5.

(d) Use your approximation up to and including the term in x^5 to find an approximate value for $arsinh\ 0.5$.

(e) Calculate the percentage error in using this approximation.

(a) Shown

(b) $x - \frac{1}{6}x^3$

(c) $\frac{3}{40}$

(d) $0.48151\ldots$

(e) 0.062% (3 d.p.)

Worked example	Your turn
Find the exact coordinates of the stationary point on the curve with equation $y = 6\cosh x - \sinh x$	Find the exact coordinates of the stationary point on the curve with equation $y = 12\cosh x - \sinh x$ $\left(\frac{1}{2}\ln\frac{13}{11}, \sqrt{143}\right)$

Worked example	Your turn
Find the first three non-zero terms of the Maclaurin series for $\sinh x$ Hence find the percentage error when this approximation is used to evaluate $\sinh 0.4$	Find the first three non-zero terms of the Maclaurin series for $\cosh x$ Hence find the percentage error when this approximation is used to evaluate $\cosh 0.2$ $1+\frac{1}{2}x^2+\frac{1}{24}x^4$ 0.0000087%