

6.3) Identities and equations

Worked example

Use definitions of $\sinh x$ and $\cosh x$ to prove that $\operatorname{sech}^2 x = 1 - \tanh^2 x$

Your turn

Use definitions of $\sinh x$ and $\cosh x$ to prove that $\cosh^2 x - \sinh^2 x = 1$

Proof

Worked example

Use definitions of $\sinh x$ and $\cosh x$ to prove that:

$$\sinh (A + B) = \sinh A \cosh B + \cosh A \sinh B$$

$$\cosh (A - B) = \cosh A \cosh B - \sinh A \sinh B$$

Your turn

Use definitions of $\sinh x$ and $\cosh x$ to prove that:

$$\sinh (A - B) = \sinh A \cosh B - \cosh A \sinh B$$

Proof

$$\cosh (A + B) = \cosh A \cosh B + \sinh A \sinh B$$

Proof

Worked example

Use definitions of $\sinh x$ and $\cosh x$ to prove that $\cosh 2x = 1 + 2 \sinh^2 x$

Your turn

Use definitions of $\sinh x$ and $\cosh x$ to prove that $\cosh 2x = 2 \cosh^2 x - 1$

Proof

Worked example

Use Osborn's rule to write down the hyperbolic identities corresponding to the trigonometric identities:

$$\cos 2x = \cos^2 x - \sin^2 x$$

Your turn

Use Osborn's rule to write down the hyperbolic identities corresponding to the trigonometric identities:

$$\cos 2x = \cos^2 x - \sin^2 x$$

$$\cosh 2x = \cosh^2 x + \sinh^2 x$$

Worked example

Given that $\sinh x = \frac{3}{5}$, find the exact value of:
 $\cosh x$

$\tanh x$

$\sinh 2x$

Your turn

Given that $\sinh x = \frac{3}{4}$, find the exact value of:
 $\cosh x$

$\frac{5}{4}$

$\tanh x$

$\frac{3}{5}$

$\sinh 2x$

$\frac{15}{8}$

Worked example

Solve for all real values of x :

$$6 \sinh x + 2 \cosh x = 7$$

Your turn

Solve for all real values of x :

$$6 \sinh x - 2 \cosh x = 7$$

$$x = \ln 4$$

Worked example

Solve for all real values of x :

$$2 \sinh^2 x - 5 \cosh x = 5$$

Your turn

Solve for all real values of x :

$$2 \cosh^2 x - 5 \sinh x = 5$$

$$x = \ln \left(-\frac{1}{2} + \frac{\sqrt{5}}{2} \right)$$

$$x = \ln(3 + \sqrt{10})$$

Worked example

Solve for all real values of x :

$$\cosh 2x - 5 \sinh x + 2 = 0$$

Your turn

Solve for all real values of x :

$$\cosh 2x - 5 \cosh x + 4 = 0$$

$$x = \ln \left(\frac{3 \pm \sqrt{5}}{2} \right), x = 0$$

Worked example

Solve the equation

$$4 \sinh 3x = 15 - 6e^{3x}$$

Give your answer in the form $\frac{1}{3} \ln k$, where k is an integer

Your turn

Solve the equation

$$3 \sinh 2x = 13 - 3e^{2x}$$

Give your answer in the form $\frac{1}{2} \ln k$, where k is an integer

$$x = \frac{1}{2} \ln 3$$

Worked example

Express $5 \cosh x + 3 \sinh x$ in the form $R \cosh(x + \alpha)$, where $R > 0$.

Give the value of α correct to 3 decimal places.

Hence write down the minimum value of $10 \cosh x + 6 \sinh x$

Your turn

Express $10 \cosh x + 6 \sinh x$ in the form $R \cosh(x + \alpha)$, where $R > 0$.

Give the value of α correct to 3 decimal places.

Hence write down the minimum value of $10 \cosh x + 6 \sinh x$

$$8 \cosh(x + 0.693)$$

$$\text{Minimum} = 8$$