

### 3. Matrix Multiplication

Examples

1.  $\begin{pmatrix} 2 & -4 \\ 3 & 8 \end{pmatrix} \begin{pmatrix} 4 \\ 6 \end{pmatrix} =$

2.

$$\begin{bmatrix} 1 & 0 & 3 & -2 \\ 2 & 8 & 4 & 3 \\ 7 & -1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 5 & 1 \\ 1 & 7 \\ 0 & 3 \\ 8 & -3 \end{bmatrix} = \begin{bmatrix} & \\ & \\ & \end{bmatrix}$$

Matrix Multiplication Involving I:



## Test Your Understanding

1.  $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \begin{pmatrix} 3 \\ -1 \end{pmatrix}$

2.  $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \begin{pmatrix} 2 & 0 & -1 \\ 3 & 2 & 1 \end{pmatrix}$

3.  $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}^2$

4.  $\begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix}^k$

5.  $(1 \ 2 \ 3) \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$

6.  $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} (1 \ 2 \ 3)$

## When is Matrix Multiplication Valid?

Matrix multiplications are not always valid: the dimensions have to agree.

- For two matrices A and B, the matrix multiplication AB is valid provided A has the same number of columns as B has rows.
- If we multiply an  $n \times m$  matrix by an  $m \times k$  matrix we generate an  $n \times k$  matrix.
- Note that only **square matrices** (i.e. same width as height) can be raised to a power.

## Properties of Matrix Operations

### **Properties of Addition**

The basic properties of addition for real numbers also hold true for matrices.

Let A, B and C be  $m \times n$  matrices

$$A + B = B + A \quad \text{commutative}$$

$$A + (B + C) = (A + B) + C \quad \text{associative}$$

### **Properties of Multiplication**

Let A, B and C be matrices of dimensions such that the following are defined. Then

$$A(BC) = (AB)C \quad \text{associative}$$

$$A(B + C) = AB + AC \quad \text{distributive}$$

$$(A + B)C = AC + BC \quad \text{distributive}$$

$$\text{But } AB \neq BA \quad \text{non - commutative}$$