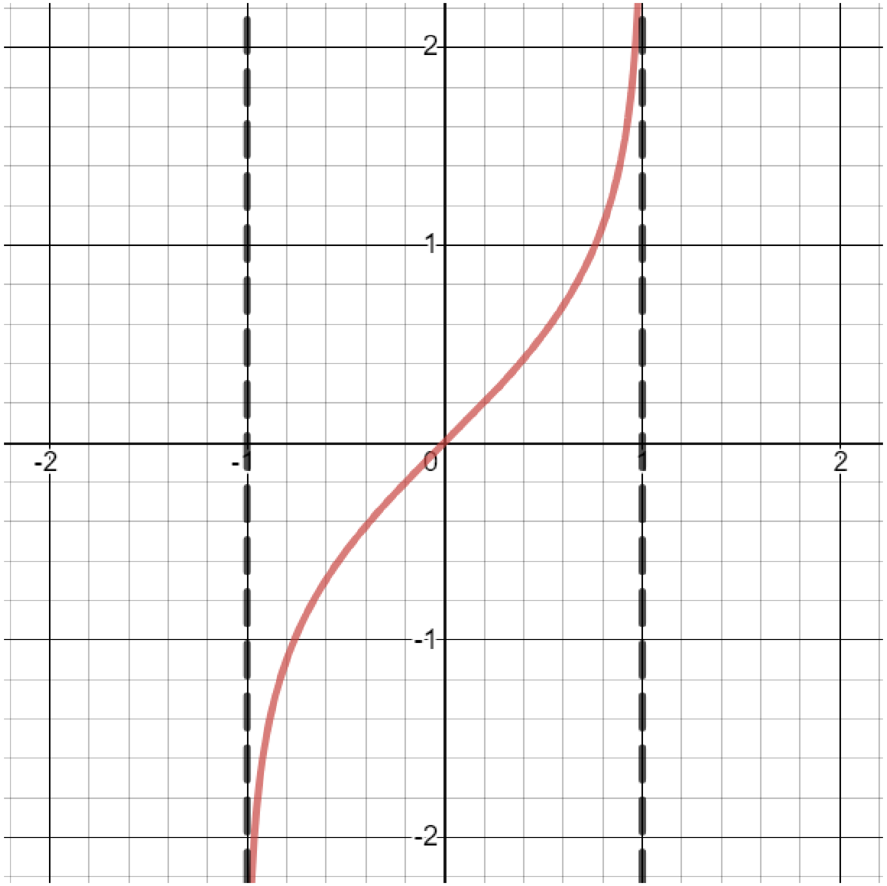
# 6.2) Inverse hyperbolic functions

| Worked example | Your turn |
|---|---|
| Sketch the graphs of: | Sketch the graphs of: |
| $y = arsinh\, x$ | $y = artanh\, x$ |
| $y = arcosh\, x$ |  |



Graphs used with permission from DESMOS: https://www.desmos.com/

| Worked example | Your turn |
| --- | --- |
| Express as natural logarithms: | Express as natural logarithms: |
| $arsinh\ 2$ | $arsinh\ 1$ |
| | $\ln\left(1+\sqrt{2}\right)$ |
| $arcosh\ 1$ | $arcosh\ 2$ |
| | $\ln(2+\sqrt{3})$ |
| $artanh\ 3$ | $artanh\ \frac{1}{3}$ |
| | $\ln\sqrt{2}$ |

| Worked example | Your turn |
|---|---|
| Prove that $arcosh\ x = \ln\left(x + \sqrt{x^2 - 1}\right), x \geq 1$ | Prove that $arsinh\ x = \ln\left(x + \sqrt{x^2 + 1}\right)$ |

$$y = arsinh\ x$$
$$x = \sinh y$$
$$x = \frac{e^y - e^{-y}}{2}$$
$$e^y - e^{-y} = 2x$$
$$e^{2y} - 2xe^y - 1 = 0$$
$$e^y = \frac{-(-2x) \pm \sqrt{(-2x)^2 - 4(1)(-1)}}{2(1)}$$
$$= x \pm \sqrt{x^2 + 1}$$

Since $\sqrt{x^2 + 1} > x$, we can only use the positive case as $e^y > 0$

$$e^y = x + \sqrt{x^2 + 1}$$
$$y = \ln\left(x + \sqrt{x^2 + 1}\right)$$
$$arsinh\ x = \ln\left(x + \sqrt{x^2 + 1}\right)$$

| Worked example | Your turn |
| --- | --- |
| Given that $artanh\ x + artanh\ y = ln\sqrt{5}$, find an expression for $y$ in terms of $x$ | Given that $artanh\ x + artanh\ y = ln\sqrt{3}$, prove that $y = \frac{2x-1}{x-2}$<br>Proof |