6A Introduction to Matrices

1. Write the dimensions of the following matrices

a) $\begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$

b) $\begin{bmatrix} 1 & 0 & 2 \end{bmatrix}$

c) $\begin{bmatrix} 4 \\ -1 \end{bmatrix}$

d) $\begin{bmatrix} 3 & 2 \\ -1 & 1 \\ 0 & -3 \end{bmatrix}$

2. Find the value of:

a) $\begin{bmatrix} 2 & -1 \\ 0 & 3 \end{bmatrix} + \begin{bmatrix} -1 & 4 \\ 5 & 3 \end{bmatrix}$

b) $\begin{bmatrix} 1 & -3 \\ 4 & 2 \\ 1 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 2 \\ 1 & -5 \\ -3 & 2 \end{bmatrix}$

3. Given that **A** + **B** = **C**, find the values of a, b, x and y

$$\boldsymbol{A} = \begin{bmatrix} 2 & 3 \\ 1 & a \end{bmatrix}, \ \boldsymbol{B} = \begin{bmatrix} b & -1 \\ 2 & 4 \end{bmatrix}, \ \boldsymbol{C} = \begin{bmatrix} 3 & y \\ x & 3 \end{bmatrix}$$

4. Given

$$\boldsymbol{A} = \begin{bmatrix} 1 & 2 \\ -1 & 0 \end{bmatrix}, \ \boldsymbol{B} = \begin{bmatrix} 6 & 0 & -4 \end{bmatrix}$$

a) Find the value of 2**A**:

b) Find the value of $^1/_2$**B**:

5. Given that **A** + 2**B** = **C**, find the values of a, b and c

$$\boldsymbol{A} = \begin{bmatrix} a & 0 \\ 1 & 2 \end{bmatrix}, \ \boldsymbol{B} = \begin{bmatrix} 1 & b \\ 0 & 3 \end{bmatrix}, \ \boldsymbol{C} = \begin{bmatrix} 6 & 6 \\ 1 & c \end{bmatrix}$$

6B Matrix Multiplication

1. Calculate the value of AB when:

   $\boldsymbol{A} = \begin{bmatrix} 1 & -2 \\ 3 & 4 \end{bmatrix}, \ \boldsymbol{B} = \begin{bmatrix} -3 \\ 2 \end{bmatrix}$

2. Given that:

   $\boldsymbol{A} = \begin{bmatrix} -1 & 0 \\ 2 & 3 \end{bmatrix}, \ \boldsymbol{B} = \begin{bmatrix} 4 & 1 \\ 0 & -2 \end{bmatrix}$

   Calculate the value of **AB** and **BA**

3. Given that:

$\boldsymbol{A} = \begin{bmatrix} 1 & -1 & 2 \end{bmatrix}$, $\boldsymbol{B} = \begin{bmatrix} 3 & -2 \end{bmatrix}$, $\boldsymbol{C} = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$

Determine whether each of the following can be evaluated and if so, find the product:

a) **AB**

b) **BC**

c) **CA**

d) **BCA**

4. Given that BA = (0), calculate AB in terms of a.

$$\boldsymbol{A} = \begin{bmatrix} -1 \\ a \end{bmatrix}, \ \boldsymbol{B} = \begin{bmatrix} b & 2 \end{bmatrix}$$

6C Determinants

Identity Matrices:

I_2

I_3

Determinants for a 2x2 Matrix

1. Given that $\boldsymbol{A} = \begin{bmatrix} 6 & 5 \\ 1 & 2 \end{bmatrix}$, find $\det \boldsymbol{A}$

2. Given that $\boldsymbol{A}$ is singular, find the value of p if (singular means det**A** = 0)

$$\boldsymbol{A} = \begin{bmatrix} 4 & p+2 \\ -1 & 3-p \end{bmatrix}$$

Determinants for a 3x3 Matrix

3. Find the minor of the element 2 in the matrix:

$$\begin{bmatrix} 5 & 0 & 2 \\ -1 & 8 & 1 \\ 6 & 7 & 3 \end{bmatrix}$$

4. Find the value of $\begin{vmatrix} 1 & 2 & 4 \\ 3 & 2 & 1 \\ -1 & 4 & 3 \end{vmatrix}$

Using a calculator to find determinants:

5. The matrix $A = \begin{bmatrix} 3 & k & 0 \\ -2 & 1 & 2 \\ 5 & 0 & k+3 \end{bmatrix}$, where k is a constant.

a) Find $\det \boldsymbol{A}$ in terms of k

b) Given that A is singular, find the possible values of k

6D Inverse of 2×2 Matrices

$\boldsymbol{A}\boldsymbol{A}^{-1}$ = I

1. For each of the matrices below, determine if they are singular and if they are not, find their inverse:

a) $\boldsymbol{A} = \begin{bmatrix} 3 & 2 \\ -1 & 1 \end{bmatrix}$

b) $\boldsymbol{B} = \begin{bmatrix} 2 & 1 \\ 2 & 1 \end{bmatrix}$

c) $\boldsymbol{C} = \begin{bmatrix} 1 & 3 \\ 2 & 0 \end{bmatrix}$

2. **A** and **B** are 2 x 2 non-singular matrices such that **BAB** = **I**.

a) Prove that **A** = $\mathbf{B}^{-1}\mathbf{B}^{-1}$

b) Given that:

$$\boldsymbol{B} = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$$

Find the matrix **A** such that **BAB** = **I**

6E Inverse of 3×3 Matrices

1. Given that the Matrix $\boldsymbol{A} = \begin{bmatrix} 1 & 3 & 1 \\ 0 & 4 & 1 \\ 2 & -1 & 0 \end{bmatrix}$, find $\boldsymbol{A}^{-1}$

2. The matrices $\boldsymbol{P}$ and $\boldsymbol{Q}$ are non-singular. Prove that $(\boldsymbol{PQ})^{-1} = \boldsymbol{Q}^{-1}\boldsymbol{P}^{-1}$.

3. The matrix $\boldsymbol{A} = \begin{bmatrix} -2 & 3 & -3 \\ 0 & 1 & 0 \\ 1 & -1 & 2 \end{bmatrix}$ and the matrix $\boldsymbol{B}$ is such that $(\boldsymbol{AB})^{-1} = \begin{bmatrix} 8 & -17 & 9 \\ -5 & 10 & -6 \\ -3 & 5 & -4 \end{bmatrix}$

a) Show that $\boldsymbol{A}^{-1} = \boldsymbol{A}$

b) Find $\boldsymbol{B}^{-1}$

6F Part 1 Solving Equations with Matrices

If: $\boldsymbol{A}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \boldsymbol{v}$

Then: $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \boldsymbol{A}^{-1}\boldsymbol{v}$

1. Use an inverse matrix to solve the simultaneous equations:

$$-x + 6y - 2z = 21$$

$$6x - 2y - z = -16$$

$$-2x + 3y + 5z = 24$$

2. A colony of 1000 mole rats is made up of adult males, adult females, and youngsters. Originally there were 100 more adult females than adult males.
After one year, the number of adult males had increased by 2%, the number of adult females had increased by 3%, the number of youngsters had decreased by 4%, and the total number of mole rats had decreased by 20.
Find out how many of each type of mole rat were originally in the colony.

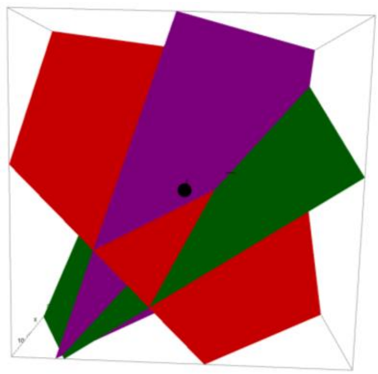
6F Part 2 Intersecting Planes

Non-Singular = one solution

$$x + y + z = 2$$

$$2x + 3y - z = 13$$

$$x - 2y + 3z = -11$$

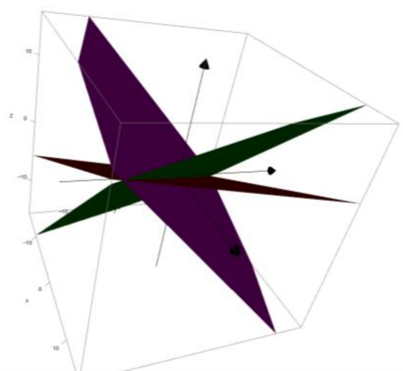


Singular: sheaf

$$3x - y - 6z = 1$$

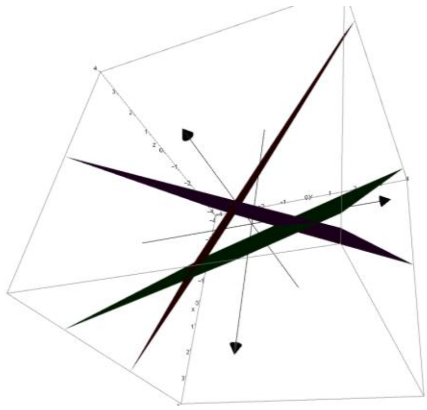
$$x + 3y + 3z = 2$$

$$-3x - y + 3z = -2$$



Singular: prism

$$3x + 6y - 6z = -6$$
$$-6x + 3y + 3z = 2$$
$$-3x - y + 3z = -2$$



Singular: parallel planes

$$x + y + z = 8$$
$$2x + 2y + 2z = 14$$
$$3x - y - z = 10$$