Core Pure 1

Volumes of Revolution

Chapter Overview

1: Find the volume when a curve is rotated around the x-axis.

2: Find the volume when a curve is rotated around the y-axis.

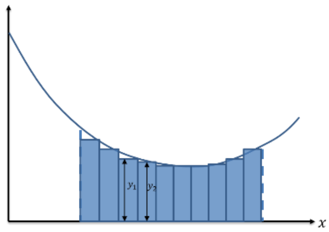
3: Find more complex volumes by adding/subtracting.

5 Further calculus	5.1	**Derive formulae for and calculate volumes of revolution.**	**Both** $\pi \int y^2 \, dx$ **and** $\pi \int x^2 \, dy$ **are required.** Students should be able to find a volume of revolution given either Cartesian equations or parametric equations.

Revolving around the x-axis

$\int_a^b y\,dx$ gives the area bounded between $y = f(x)$, $x = a$, $x = b$ and the x-axis.

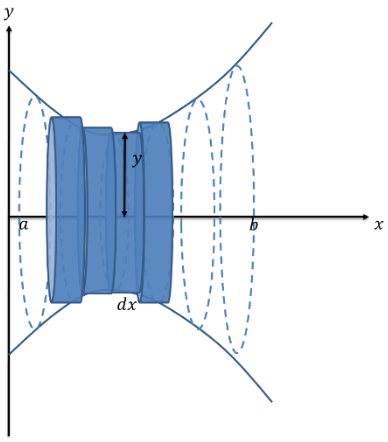
If we split up the area into thin rectangular strips, each with width dx and each with height the $y = f(x)$ for that particular value of x. Each has area $f(x) \times\ dx$.



If we had 'discrete' strips, the total area would be:

$$\Sigma_{x=a}^{b}\ (f(x)\ dx)$$

But because the strips are infinitely small and we have to think continuously, we use $\int$ instead of Σ.
Integration therefore can be thought of as a continuous version of summation.



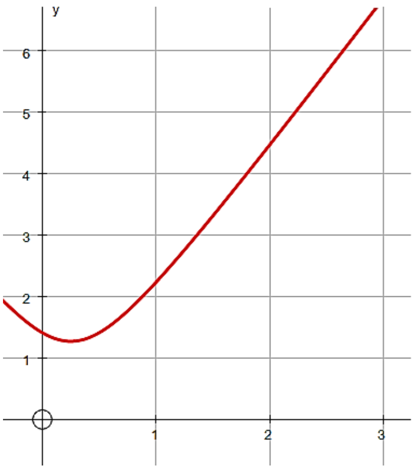
Now suppose we spun the line $y = f(x)$ about the x axis to form a solid (known as a *volume of revolution*)

Examples

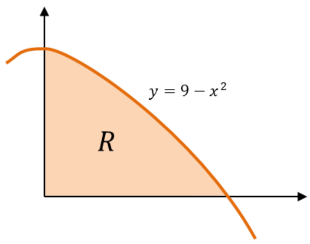
1. The region R is bounded by the y-axis, the curve with equation

$$y = \sqrt{(6x^2 - 3x + 2)}$$

and the lines x = 1 and x = 2 . The region is rotated through $360°$ about the x-axis. Find the exact volume of the solid generated.



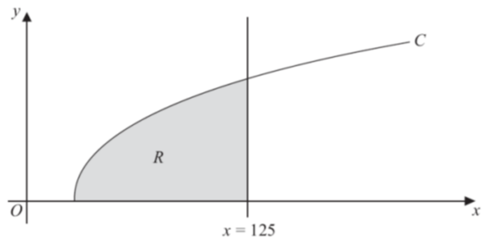
2. The diagram shows the region R which is bounded by the x-axis, the y-axis and the curve with equation $y = 9 - x^2$. The region is rotated through $360°$ about the x-axis. Find the exact volume of the solid generated.



Test Your Understanding

$$y = \left(x^{\frac{2}{3}} - 9\right)^{\frac{1}{2}}$$

The finite region R which is bounded by the curve C, the x-axis and the line $x = 125$ is shown shaded in Figure 3. This region is rotated through 360° about the x-axis to form a solid of revolution.



Use calculus to find the exact value of the volume of the solid of revolution. **(5)**

Ex 5a Pg 73-75