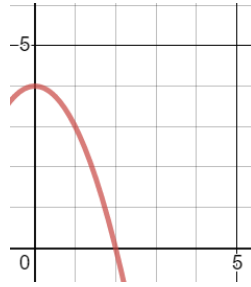


## 5.1) Volumes of revolution around the $x$ -axis

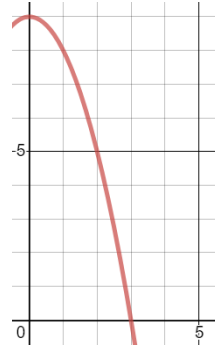
## Worked example

A sketch of  $y = 4 - x^2$  is shown. The region  $R$  is bounded by the  $x$ -axis, the  $y$ -axis and the curve with equation  $y = 4 - x^2$ . The region is rotated through  $360^\circ$  about the  $x$ -axis. Find the exact volume of the solid generated.



## Your turn

A sketch of  $y = 9 - x^2$  is shown. The region  $R$  is bounded by the  $x$ -axis, the  $y$ -axis and the curve with equation  $y = 9 - x^2$ . The region is rotated through  $360^\circ$  about the  $x$ -axis. Find the exact volume of the solid generated.



$$\frac{648\pi}{5}$$

## Worked example

Find the exact volume of the solid generated when the curve is rotated through  $360^\circ$  about the  $x$ -axis between the given limits:

$$y = 1 - \frac{1}{x^2} \text{ between } x = 1 \text{ and } x = 4$$

## Your turn

Find the exact volume of the solid generated when the curve is rotated through  $360^\circ$  about the  $x$ -axis between the given limits:

$$y = 1 + \frac{1}{x^2} \text{ between } x = 1 \text{ and } x = 2$$

$$\frac{55}{24}\pi$$

## Worked example

A finite region is bounded by the curve with equation  $y = (x^{\frac{3}{2}} - 8)^{\frac{1}{2}}$ , the  $x$ -axis and the line  $x = 9$ . This region is rotated  $360^\circ$  about the  $x$ -axis to form a solid of revolution. Find the exact value of the volume of the solid of revolution

## Your turn

A finite region is bounded by the curve with equation  $y = (x^{\frac{2}{3}} - 9)^{\frac{1}{2}}$ , the  $x$ -axis and the line  $x = 125$ . This region is rotated  $360^\circ$  about the  $x$ -axis to form a solid of revolution. Find the exact value of the volume of the solid of revolution

$$\frac{4236\pi}{5}$$

## Worked example

A curve has equation  $7y^2 - x^3 = 2x - 12$ . A finite region is bounded by the curve, the  $x$ -axis and the line  $x = 5$ . The region is rotated about the  $x$ -axis to generate a solid of revolution. Find the volume of the solid generated.

## Your turn

A curve has equation  $5y^2 - x^3 = 2x - 3$ . A finite region is bounded by the curve, the  $x$ -axis and the line  $x = 4$ . The region is rotated about the  $x$ -axis to generate a solid of revolution. Find the volume of the solid generated.

$$\frac{279}{20}\pi$$

## Worked example

A curve has equation  $y = x\sqrt{9 - x^2}$ . A finite region is bounded by the curve, the  $x$ -axis and the line  $x = a$  where  $0 < a < 3$ . The region is rotated through  $2\pi$  radians to generate a solid of revolution with volume  $\frac{1025\pi}{32}$ . Find the value of  $a$

## Your turn

A curve has equation  $y = x\sqrt{4 - x^2}$ . A finite region is bounded by the curve, the  $x$ -axis and the line  $x = a$  where  $0 < a < 2$ . The region is rotated through  $2\pi$  radians to generate a solid of revolution with volume  $\frac{657\pi}{160}$ . Find the value of  $a$

$$a = \frac{1}{2}$$