5.1) Polar coordinates and equations

Worked example

Convert from Cartesian to polar coordinates:

$(4, -3)$

$(-5, 12)$

$(\sqrt{3}, 1)$

Your turn

Convert from Cartesian to polar coordinates:

$(3, 4)$

$(5, 0.927)$

$(5, -12)$

$(13, -1.176)$

$(-\sqrt{3}, -1)$

$\left(2, \frac{7\pi}{6}\right)$ or $\left(2, -\frac{5\pi}{6}\right)$

Worked example

Convert from polar to Cartesian coordinates:

$$\left(8, \frac{-5\pi}{3}\right)$$

$$\left(4, \frac{\pi}{3}\right)$$

$$(3, \frac{\pi}{2})$$

Your turn

Convert from polar to Cartesian coordinates:

$$\left(10, \frac{4\pi}{3}\right)$$

$$(-5, 5\sqrt{3})$$

$$\left(8, \frac{2\pi}{3}\right)$$

$$(-4, 4\sqrt{3})$$

$$(2, \pi)$$

$$(-2, 0)$$

Worked example

Find Cartesian equations for the following curves:

$r = 4$

$r = 3 + \cos 4\theta$

$r^4 = \sin 2\,\theta, \qquad 0 < \theta \leq \frac{\pi}{2}$

Your turn

Find Cartesian equations for the following curves:

$r = 5$

$x^2 + y^2 = 25$

$r = 2 + \cos 2\theta$

$(x^2 + y^2)^{\frac{3}{2}} = 3x^2 + y^2$

$r^2 = \sin 2\,\theta, \qquad 0 < \theta \leq \frac{\pi}{2}$

$(x^2 + y^2)^2 = 2xy$

Worked example

Find Cartesian equations for the following curves:

$r = 5\sec\theta$

$r = 3cosec\ \theta$

$r = 4\cos\theta$

$r = 2\sin\theta$

Your turn

Find Cartesian equations for the following curves:

$r = 3\sec\theta$

$x = 3$

$r = 5cosec\ \theta$

$y = 5$

$r = 2\cos\theta$

$x^2 + y^2 = 2x$ or $(x-1)^2 + y^2 = 1$

$r = 4\sin\theta$

$x^2 + y^2 = 4y$ or $x^2 + (y-2)^2 = 4$

Worked example

Find Cartesian equations for the following curves:

$r = 8 \cot\theta \operatorname{cosec}\theta$

$r^2 = 1 + \cot^2\theta$

Your turn

Find Cartesian equations for the following curves:

$r = 4 \tan\theta \sec\theta$

$x^2 = 4y$ or $y = \frac{x^2}{4}$

$r^2 = 1 + \tan^2\theta$

$x^2 = 1$ or $x = \pm 1$

Worked example

Find polar equations for the following curves:

$y^2 = 2x$

$x^2 - y^2 = 10$

$y\sqrt{2} = x + 8$

Your turn

Find polar equations for the following curves:

$y^2 = 4x$

$r = 4 \cot\theta \, cosec\, \theta$

$x^2 - y^2 = 5$

$r^2 = 5 \sec 2\theta$

$y\sqrt{3} = x + 4$

$r = 2cosec\left(\theta - \frac{\pi}{6}\right)$

Worked example

Find polar equations for the following curves:

$y = 4x$

$xy = 8$

$y = -\sqrt{2}x + 4$

Your turn

Find polar equations for the following curves:

$y = 2x$

$\tan\theta = 2$

$xy = 4$

$r^2 = 8\ cosec\ 2\theta$

$y = -\sqrt{3}x + 4$

$r = 2\ cosec\left(\theta + \frac{\pi}{3}\right)$

Worked example

Find polar equations for the following curves:

$$x^2 + y^2 - 4x = 0$$

$$(x + y)^2 = 8$$

$$x - y = 5$$

Your turn

Find polar equations for the following curves:

$$x^2 + y^2 - 2x = 0$$

$$r = 2\cos\theta$$

$$(x + y)^2 = 4$$

$$r^2 = \frac{4}{1 + \sin 2\theta}$$

$$x - y = 3$$

$$r = \frac{3}{\sqrt{2}}\sec\left(\theta + \frac{\pi}{4}\right)$$