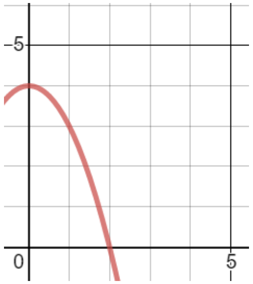
5) Volumes of revolution

5.1) Volumes of revolution around the x-axis

Chapter CONTENTS

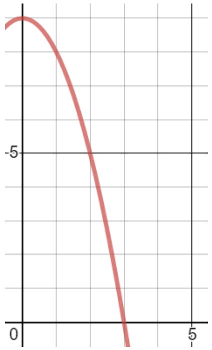
Worked example

A sketch of $y = 4 - x^2$ is shown. The region R is bounded by the x-axis, the y-axis and the curve with equation $y = 4 - x^2$. The region is rotated through 360° about the x-axis. Find the exact volume of the solid generated.



Your turn

A sketch of $y = 9 - x^2$ is shown. The region R is bounded by the x-axis, the y-axis and the curve with equation $y = 9 - x^2$. The region is rotated through 360° about the x-axis. Find the exact volume of the solid generated.



$$\frac{648\pi}{5}$$

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Worked example	Your turn
Find the exact volume of the solid generated when the curve is rotated through 360° about the x-axis between the given limits: $y = 1 - \frac{1}{x^2}$ between $x = 1$ and $x = 4$	Find the exact volume of the solid generated when the curve is rotated through 360° about the x-axis between the given limits: $y = 1 + \frac{1}{x^2}$ between $x = 1$ and $x = 2$ $\frac{55}{24}\pi$

Worked example	Your turn
A finite region is bounded by the curve with equation $y = \left(x^{\frac{3}{2}} - 8\right)^{\frac{1}{2}}$, the x-axis and the line $x = 9$. This region is rotated $360°$ about the x-axis to form a solid of revolution. Find the exact value of the volume of the solid of revolution	A finite region is bounded by the curve with equation $y = \left(x^{\frac{2}{3}} - 9\right)^{\frac{1}{2}}$, the x-axis and the line $x = 125$. This region is rotated $360°$ about the x-axis to form a solid of revolution. Find the exact value of the volume of the solid of revolution $\frac{4236\pi}{5}$

Worked example	Your turn
A curve has equation $7y^2 - x^3 = 2x - 12$. A finite region is bounded by the curve, the x-axis and the line $x = 5$. The region is rotated about the x-axis to generate a solid of revolution. Find the volume of the solid generated.	A curve has equation $5y^2 - x^3 = 2x - 3$. A finite region is bounded by the curve, the x-axis and the line $x = 4$. The region is rotated about the x-axis to generate a solid of revolution. Find the volume of the solid generated. $\frac{279}{20}\pi$

Worked example

A curve has equation $y = x\sqrt{9 - x^2}$. A finite region is bounded by the curve, the x-axis and the line $x = a$ where $0 < a < 3$. The region is rotated through 2π radians to generate a solid of revolution with volume $\frac{1025\pi}{32}$. Find the value of a

Your turn

A curve has equation $y = x\sqrt{4 - x^2}$. A finite region is bounded by the curve, the x-axis and the line $x = a$ where $0 < a < 2$. The region is rotated through 2π radians to generate a solid of revolution with volume $\frac{657\pi}{160}$. Find the value of a

$$a = \frac{1}{2}$$

5.2) Volumes of revolution around the y-axis

Worked example	Your turn
A curve has equation $y = \sqrt{x-2}$. A finite region is bounded by the curve, the y-axis and the lines $y = 1$ and $y = 4$. The region is rotated through $360°$ about the y-axis. Find the volume of the solid generated.	A curve has equation $y = \sqrt{x-1}$. A finite region is bounded by the curve, the y-axis and the lines $y = 1$ and $y = 3$. The region is rotated through $360°$ about the y-axis. Find the volume of the solid generated. $\frac{1016\pi}{15}$

Worked example	Your turn
A curve has equation $y = \sqrt[3]{3x + 1}$. A finite region is bounded by the curve, the y-axis and the lines $y = 2$ and $y = 5$. The region is rotated through $360°$ about the y-axis. Find the volume of the solid generated.	A curve has equation $y = \sqrt[3]{2x + 1}$. A finite region is bounded by the curve, the y-axis and the lines $y = 2$ and $y = 4$. The region is rotated through $360°$ about the y-axis. Find the volume of the solid generated. $\frac{7715\pi}{14}$

Worked example	Your turn
A curve has equation $x = y^2 - 4y + 8$. A finite region is bounded by the curve, the y-axis and the lines $y = 1$ and $y = 5$. a) Find the area of the region b) The region is rotated through $360°$ about the y-axis. Find the volume of the solid generated.	A curve has equation $x = y^2 - 6y + 10$. A finite region is bounded by the curve, the y-axis and the lines $y = 1$ and $y = 4$. a) Find the area of the region b) The region is rotated through $360°$ about the y-axis. Find the volume of the solid generated. a) 6 b) $\frac{78}{5}\pi$

Worked example	Your turn
$f(x) = x^2 - 6x + 9, x \geq 3$ A finite region is bounded by the curve $y = f(x)$, the y-axis and the lines $y = 1$ and $y = 4$ The region is rotated through 2π radians about the y-axis. Find the exact volume of the solid generated.	$f(x) = x^2 - 2x + 1, x \geq 1$ A finite region is bounded by the curve $y = f(x)$, the y-axis and the lines $y = 1$ and $y = 9$ The region is rotated through 2π radians about the y-axis. Find the exact volume of the solid generated. $\frac{248}{3}\pi$

Worked example

A curve has equation $y^2 = \frac{1}{3x+2}$

A finite region is bounded by the curve $y = f(x)$, the y-axis and the line $y = 5$

The region is rotated through 2π radians about the y-axis. Find the volume of the solid generated.

Your turn

A curve has equation $y^2 = \frac{1}{2x+1}$

A finite region is bounded by the curve $y = f(x)$, the y-axis and the line $y = 4$

The region is rotated through 2π radians about the y-axis. Find the volume of the solid generated.

$$\frac{117}{256}\pi$$

5.3) Adding and subtracting volumes

Worked example	Your turn
A finite region is bounded by the curve with equation $y = x^3 + 1$, the line $y = 3 - x$ and the x and y-axes. A solid is created by rotating the region $360°$ about the x-axis. Find the volume of this solid.	A finite region is bounded by the curve with equation $y = x^3 + 2$, the line $y = 5 - 2x$ and the x and y-axes. A solid is created by rotating the region $360°$ about the x-axis. Find the volume of this solid. $\frac{135\pi}{14}$

Worked example	Your turn
A finite region is bounded by the curves with equations $y = \sqrt{x}$ and $y = \frac{1}{27x}$ and the line $x = 2$. The region is rotated through $360°$ about the x-axis. Find the exact volume of the solid generated.	A finite region is bounded by the curves with equations $y = \sqrt{x}$ and $y = \frac{1}{8x}$ and the line $x = 1$. The region is rotated through $360°$ about the x-axis. Find the exact volume of the solid generated. $\frac{27\pi}{64}$

Worked example	Your turn
The area between the lines with equations $y = x$ and $y = \sqrt{x}$, where $x \geq 0$ is rotated $360°$ about the x-axis. Determine the volume of the solid generated.	The area between the lines with equations $y = x$ and $y = \sqrt[3]{x}$, where $x \geq 0$ is rotated $360°$ about the x-axis. Determine the volume of the solid generated. $\frac{4\pi}{15}$

5.4) Modelling with volumes of revolution

Chapter CONTENTS

Worked example

A manufacturer wants to cast a prototype for a new design for a lightbulb out of glass. A region is used as a model for the cross-section of the lightbulb. The region is bounded by the x-axis and the curve with equation $y = k - 60x^2$, and will be rotated around the y-axis. Each unit on the coordinate axes represents 1cm.

(a) Suggest a suitable value for k.

(b) Use your value of k to estimate the volume of glass needed to make the prototype.

(c) State one limitation of this model.

Your turn

A manufacturer wants to cast a prototype for a new design for a pen barrel out of solid resin. A region is used as a model for the cross-section of the pen barrel. The region is bounded by the x-axis and the curve with equation $y = k - 100x^2$, and will be rotated around the y-axis. Each unit on the coordinate axes represents 1cm.

(a) Suggest a suitable value for k.

(b) Use your value of k to estimate the volume of resin needed to make the prototype.

(c) State one limitation of this model.

(a) $k = 10$ $(10 \leq k \leq 15$ sensible)

(b) $1.57cm^3$ (3 sf)

(c) The cross-section of the pen unlikely to match the curve exactly

Worked example	Your turn
Use integration to show that the volume of a cylinder is $V = \pi r^2 h$	Use integration to show that the volume of a cone is $V = \frac{1}{3}\pi r^2 h$ Shown
Use integration to show that the volume of a sphere is $V = \frac{4}{3}\pi r^3$	